Engineering Structures 145 (2017) 34–43

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

A weak-form quadrature element formulation for 3D beam elements used in nonlinear and postbuckling analysis of space frames

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ARTICLE INFO

Article history:
Received 1 February 2017
Revised 30 March 2017
Accepted 4 May 2017

Keywords:
Space frame
Quadrature element
Geometric nonlinearity
Postbuckling
Weak form

ABSTRACT

A weak-form quadrature element formulation is presented for the three-dimensional beam element for use in the geometrically nonlinear and postbuckling analysis of space frames. Starting from the virtual work equation of a beam in the linearized, incremental sense, the quadrature element method (QEM) is employed to derive the elastic stiffness, geometric stiffness, and induced moment matrices of the beam with due account taken of the large rotations in three-dimensional space. All the stiffness matrices are adopted in the incremental-iterative analysis using the generalized displacement control (GDC) method, with specific considerations for the predictor and corrector phases. By comparing the results obtained for all the benchmark problems studied with existing ones, it is demonstrated that the present formulation is capable of predicting large displacements and rotations, as well as the postbuckling paths of space frames. The present formulation is featured by the fact that it is simple, straightforward, and reliable.

1. Introduction

Geometrically nonlinear and postbuckling behaviors of space frames have attracted widespread attention of researchers and engineers over the past few decades due to the advancement of engineering design. Developments of new formulation theories and novel computer methods are two major aspects of the subject. Yang et al. [1] has made a literature review concerning the various aspects of geometrically and postbuckling analysis of space frames. Besides the beam-column theory [2,3] from the perspective of conventional structural mechanics, nonlinear continuum mechanics-based beam theories [4] have gained more and more interest.

In general, two different formulations, i.e., the total Lagrangian (TL) formulation and updated Lagrangian (UL) formulation, have been commonly used for the derivation of beam elements [5]. The former describes the current unknown configuration of a beam element based on the reference coordinates of the initial undeformed configuration, such as the geometrically exact beam theory [6–10]; while the latter describes the current configuration of a beam element based on the reference coordinates of the latest known configuration. Yang et al. [11,12] have formulated an incremental nonlinear beam theory based on the UL formulation. Another formulation based on the co-rotational (CR) description is worthy of special mentioning. Either a TL [13–16] or UL [17,18] formulation can be employed for the CR formulation. The CR formulation is capable of taking the initially non-straight configuration of the element into account.

Because of the geometric nonlinearity, an incremental-iterative analysis scheme is usually needed for the solution of space frames under proportionally increasing loadings. The finite element method (FEM) is used almost exclusively for solving the deformation of the structure in each iterative step. Previously, the Newton-Raphson method, arc-length method, and some other variants, have often been used for the incremental-iterative analysis of structures. As for the postbuckling analysis of a structure, the key concern is how to get around with the limit points, snap-back points, and multi winding curves. The *generalized displacement control* (GDC) method proposed by Yang et al. [19–21] can deal with all these problems in a self-adaptive manner. For this reason, it has received a continuously increasing number of users due to its appealing and reliable features for incremental-iterative analysis [22].

Recently, a novel computational method, named as the weak-form quadrature element method (QEM) [23], has been developed. The QEM starts from the weak form description of the problem, so it has the same wide applicability as the conventional FEM. With the conventional FEM with displacement-based formulation, one first converts the governing equations of an element to energy

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http://dx.doi.org/10.1016/j.engstruct.2017.05.007
0141-0296/

expressions, uses the shape functions to relate field parameters to nodal parameters, and then integrates over the domain to obtain the stiffness matrices. With the QEM, a different logistics is adopted for the derivation of element stiffness matrices. It first discretizes the integrands in the weak-form expressions by the Lagrange interpolation, and then approximates the differentiation at a discrete node by the differential quadrature method (DQM) [24]. The DQM approximates the differentiation of a function at a given node by a weighted linear summation of the function values at all discrete nodes in the domain. As a result, the QEM does not require additionally constructing shape functions each time for higher-order elements and the following derivation is quite straightforward, reducing the formulation complexity.

Meanwhile, due to the feature of the QEM, the numbers of the integration and differentiation nodes (they are the same set of nodes which is different from the FEM) are not fixed and they can be increased gradually to satisfy the convergence requirement. The entire converging process can be programmed for execution automatically, making the method having an inherent self-adaptivity nature.

In this paper, the QEM, instead of the FEM, is employed to investigate the geometrically nonlinear and postbuckling behaviors of space frames. In the FEM, choosing the proper form and order of the shape functions has always been a concern. With the QEM, however, the interpolation order can be increased easily and automatically, and the required order can be determined by comparison of results obtained with those of lower-order. Although Zhong et al. [25,26] have used the QEM to study similar problems, their formulations are based on the TL formulation of the geometrically exact beam theory in combination with the conventional Newton-Raphson iterative method. In the present work, the UL formulation-based incremental nonlinear beam theory in conjunction with the GDC method, both are proposed by Yang et al., is to be adopted. This formulation enjoys the benefits of simplicity, straightforwardness, and reliability.

2. Formulation

2.1. Incremental virtual work equation

The QEM starts from the weak-form description of a system, so an energy expression, instead of a governing differential equation of motion, of the geometrically nonlinear beam element needs to be formulated first. By selecting the latest known configuration as the reference configuration, a general incremental equation of equilibrium in a linearized sense derived from the virtual work principle is given by [5,11]

$$\delta U + \delta V = \int_{\Omega} C_{ijkl} e_{kl} \delta e_{ij} d\Omega + \int_{\Omega} \tau_{ij} \delta \eta_{ij} d\Omega = {}^2R - {}^1R \tag{1}$$

where δ denotes the variation, U is the strain energy, V is the potential energy of the initial stresses existing on the element at the latest known configuration, Ω denotes the volume of the latest known configuration, C_{ijkl} is the constitutive coefficient, τ_{ij} is the Cauchy stress, e_{ij} and η_{ij} are the linear and nonlinear components of the corresponding Green-Lagrange strain increment, and ${}^1R$ and ${}^2R$ are the virtual works done by the nodal forces acting on the element at the latest known and current configurations, respectively. According to the nonlinear continuum mechanics, e_{ij} and η_{ij} are defined by

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2}$$

$$\eta_{ij} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right) \tag{3}$$

where u_i is the displacement increment and x_i is the coordinate of the element at the latest known configuration.

For the three-dimensional rectangular beam element shown in Fig. 1, the displacements (u_1, u_2, u_3) of a generic point can be related to the displacements (u, v, w) of the centroid of the same section as follows [11]:

$$u_1 = u - yv' - zw' \tag{4}$$

$$u_2 = v - z\theta_x \tag{5}$$

$$u_3 = w + y\theta_x \tag{6}$$

where θ_x is the torsional angle. The superscript "prime" indicates the first-order partial differentiation with respect to the x-axis.

Substituting Eqs. (4)–(6) and the constitutive relation for elasticity into Eqs. (1)–(3), the variation of the strain energy, U, can be expressed as [11]

$$\delta U = \frac{1}{2}\int_0^L \left(EA\delta u'^2 + EI_z\delta v''^2 + EI_y\delta w''^2 + GJ\delta\theta_x'^2\right)dx \tag{7}$$

where L is the beam length, A is the cross-sectional area, E is the Young's modulus, G is the shear modulus, I_y and I_z are the area moments of inertia about the y- and z-axis, respectively, and J is the torsion constant. The superscript "double prime" indicates the second-order partial differentiation with respect to the x-axis.

The potential energy, V, of the initial stresses acting on the element at the latest known configuration is contributed by four components, namely, the axial force ${}^1F_x$, shear forces ${}^1F_y$ and ${}^1F_z$, bending moments ${}^1M_y$ and ${}^1M_z$, and torsional moment ${}^1M_x$. Therefore, the variation of the potential energy, δV, can be divided into four parts as follows:

$$\delta V = \delta V_1 + \delta V_2 + \delta V_3 + \delta V_4 \tag{8}$$

where

$$\delta V_1 = \frac{1}{2}\int_0^L \left[{}^1F_x\delta\left(u'^2 + v'^2 + w'^2\right) + {}^1F_x\left(\frac{I_z}{A}\delta v''^2 + \frac{I_y}{A}\delta w''^2\right) + {}^1F_x\frac{I_y + I_z}{A}\delta\theta_x'^2\right]dx \tag{9}$$

$$\delta V_2 = \int_0^L \left[{}^1F_y\delta(w'\theta_x - u'v') - {}^1F_z\delta(v'\theta_x + u'w')\right]dx \tag{10}$$

$$\delta V_3 = \int_0^L \left[-{}^1M_z\delta\left(w'\theta_x'\right) - {}^1M_y\delta\left(v'\theta_x'\right) - {}^1M_y\delta(u'w'') + {}^1M_z\delta(u'v'')\right]dx \tag{11}$$

$$\delta V_4 = \frac{1}{2}\int_0^L \left[{}^1M_x\delta(v''w') - {}^1M_x\delta(v'w'')\right]dx \tag{12}$$

The virtual works ${}^1R$ and ${}^2R$ denote the products of the nodal forces and their corresponding virtual displacements. After discretizing the beam element by N nodes, as shown in Fig. 1, they can be expressed as

$${}^2R - {}^1R = \delta\mathbf{d}^{\mathrm{T}}\left({}^2\mathbf{f} - {}^1\mathbf{f}\right) + \delta W \tag{13}$$

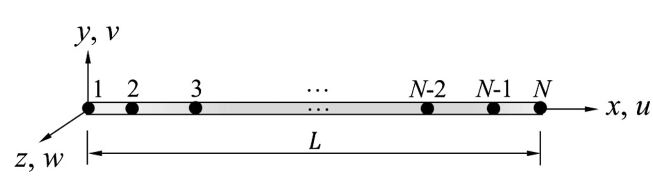

Fig. 1. Coordinate system and discretization of a beam element.