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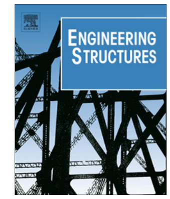

Distortional analysis of simply supported box girders with inner diaphragms considering shear deformation of diaphragms using initial parameter method

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A B S T R A C T

In this paper, the distortion of simply supported girders with inner diaphragms subjected to concentrated eccentric loads is investigated using initial parameter method (IPM), in which the in-plane shear deformation of diaphragms is fully considered. A statically indeterminate structure was modeled with inner redundant forces, where the interactions between the girder and diaphragms were indicated by a distortional moment. Considering the compatibility condition between the girder and diaphragms, solutions for the distortional angle, warping displacements and stresses were derived and further simplified by establishing a matrix equation system. The validity of IPM was intensively verified by a finite element analysis and distortional experiments. Parametric studies were then performed to examine the effect of the diaphragm number on the distortional angle, warping displacements and stresses under various ratios of height to span of the girder and the diaphragm thicknesses. Besides, stabilities of the local web plate and mid-span diaphragm were analyzed based on IPM for box girders with symmetrical three inner diaphragms. Results show that the local web plate will buckle before overall yielding with the increment of the eccentric loads P_j, and the mid-span diaphragm is constantly stable in the whole deformation process. It shows that more attentions should be paid on the stability of the local web plate than overall yielding for girders subjected to eccentric loads.

1. Introduction

During the past several decades, box girders have been widely applied in buildings and bridges due to their large bending and torsional stiffness. However, they are generally susceptible to the cross-sectional distortion [1] under eccentric loads due to their quadrilateral instability. Therefore, excessive distortional warping and transversal bending stresses will be produced besides the torsional and bending ones in box girders. In a special case, the distortional warping stresses may be significant to the torsional and bending ones. In order to control the distortion, diaphragms are installed at the interior of the girder, which can increase not only the stability of the local plate, but also the resistance to the warping deformations and stresses [2–4].

Researches on the distortion of girders with inner diaphragms have been performed for many decades. The distortion of box girder was initially studied by Dabrowski [5] who first formulated the distortion of box girders with a symmetrical cross section. Li [6,7] proposed that the shear strain of the cross section cannot be ignored when the distortional shear rigidity is significant compared to the distortional warping one for box girders. Wright [8] proposed the Beam on Elastic Foundation (BEF) analogy for the distortion of girders with inner diaphragms, where the diaphragms are analogous to inner supports. Based on BEF, Hsu [9,10] proposed the Equivalent Beam on Elastic Foundation (EBEF) analogy considering the shear strain of the cross section, and found that the EBEF analogy is more versatile than BEF due to its simplicity in analyzing more complex problems such as non-uniform sections and multi-span beams.

Interactions between the girder and diaphragms is the key issue for the distortion of girders with inner diaphragms. A statically indeterminate structure [11] was modeled with redundant forces acting along the junctions between the girder and diaphragms.

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Moreover, the force method was applied to calculate redundant forces, where elements in the stiffness matrix were obtained from the finite strip method [12]. The numerical results were then extended to multi-span bridges [13] and long-span curved bridges [14]. An outstanding contribution was made by Suetake [15], where the girder was regarded as an assembly of thin plates, and the extended trigonometric series method (ETS) was applied to analyze the stresses and deformations for box girders. Comparisons with FEM results show that ETS has a high accuracy. However, it is inconvenient to apply since there are many simultaneous nonlinear equations to solve even for girders with few diaphragms, e.g. there are up to 720 equations for a girder with two diaphragms.

The wall thickness of diaphragms and the number of diaphragms in a girder will make a significant influence on the displacements and stresses. Park [16,17] proposed a new beam element with nine degrees of freedom per node for girders. Studies showed that the distortional warping and transversal bending stresses were reduced by increasing the diaphragm number. Similar conclusions can be drawn for straight multi-cell box girders with diaphragms [18,19]. For horizontally curved bridges, the rational spacing between adjacent diaphragms was provided [20] according to the ratio between the distortional warping stress and the bending stress. Using FEM, Zhang [21] found that the rational number for diaphragms is 3 to 5 when the ratio of width to height of the cross section is 1.5 and the rational number is 9 when the ratio is 4.5. Li [22] proposed a new finite element solution, and found that the distortional warping stress for cantilever girders can be ignored when the spacing between adjacent diaphragms is less than one fifth of the span; while for simply supported and fixed girders, the spacing is less than one eighth of the span.

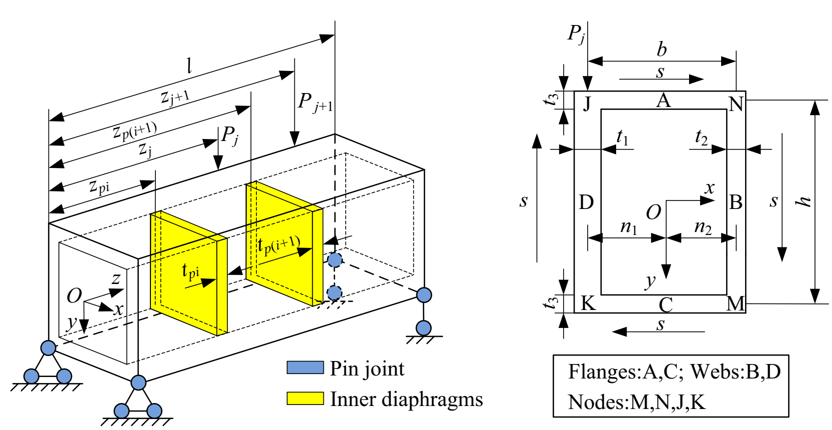
Initial parameter method (IPM) was proposed first by Vlasov [23] to analyze the non-uniform torsion of beams. Analogous to IPM in non-uniform torsion, IPM can be extended to analyze distortions of girders. Considering the effect of shear strains of the cross section, Xu [24,25] developed an equation with the variable distortional angle, and established two categories of IPMs of the fourth order, classified by the ratio of the distortional stiffness. Harashima [26] proposed a distortional equation with a distortional warping function, and established the fifth-order IPM. Both IPMs in distortion have a high efficiency compared with FEM. However, IPMs has not been extended into the distortion for girders with inner diaphragms.

For distortions of a girder with inner diaphragms, an assumption of infinite-rigidity diaphragm was generally made in most studies [16,20,27], where the in-plane deformation of diaphragms was totally restrained and warping was free. Similar assumptions can be found in distortion of curved box beams [28], where the distortional angle at the location of diaphragms is set as zero. However, the infinite-rigidity assumption is just an approximation, which is not applicable to thin flexible diaphragms. The main objective of this work is to investigate the distortion of simply supported girders with inner flexible diaphragms under concentrated eccentric loads, where the in-plane shear deformation of diaphragms is fully considered. Interactions between the girder and diaphragms are indicated by a distortional moment. Based on the compatibility condition between the girder and diaphragms, solutions for both the distortional angle and warping function are obtained from the IPM. Taking a simply supported girder with 2, 5 and 9 diaphragms, respectively, as an example, the distortional solutions from IPM were obtained, then verified by a FE analysis and experiments. This was followed by a parametric study, in which distortional deformations and stresses were investigated in terms of the diaphragm number and thickness and the height to span ratio of the girder. Based on the proposed IPM, stabilities of both the local web plate and mid-span diaphragm were examined for girders with three symmetrical inner diaphragms. A series of curves were obtained for the relations between the critical buckling load and the position of diaphragms under various height to width ratios of the cross section.

2. Structural model

Consider a simply supported box girder with inner diaphragms under concentrated eccentric loads P_j ($j = 1,2,\ldots,m$). The coordinate system *O-xyz* is illustrated in Fig. 1a with the original *O* at the shear center on one end of the girder. For analysis, the distances between *O* and the mid-lines of webs B and D are marked by n_1 and n_2 in Fig. 1b, respectively. The girder is made of a homogeneous, isotropic and linearly elastic material with its Young's and shear moduli denoted by *E* and *G*, respectively. The girder span is *l*. The thicknesses of web B and D are t_1 and t_2 and their height is *h*, while the width of flanges A and C is *b* and the thickness t_3. The thickness for the *i*th diaphragm is t_{pi} ($i = 1,2,\ldots,n$) and its mid line is marked by z_{pi}, measured from *O*. The load P_j is located on the top of web D at z_j.

Fig. 2a shows that the eccentric load P_j can be decomposed into three components [29,30] – the flexural load, the torsional and the



(a) simply supported box girder with inner diaphragms

(b) cross section

Fig. 1. Girder with inner diaphragms under eccentric loads P_j ($j = 1,2,\ldots,m$).