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Mechanical model for determining the critical load of plane frames with semi-rigid joints subjected to static loads

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ABSTRACT

The present work deals with the effect of beam-column joint flexibility on the elastic buckling load of plane steel frames. A simple and effective mechanical model is proposed and the corresponding stiffness matrix is presented. The model consists in the development of comprehensive approach taking into account, simultaneously, the effects of the joint rigidity, the elastic buckling load, and this for both sway and non-sway frames. As has been shown by previous research, only one element is required over the length of the element to model stability. This is a marked contribution and advantage of the proposed method, as well as its simplicity, and yet accuracy, to solve practical problem with little computational effort. Also, it includes stability functions in the stiffness matrix, something very often ignored by researchers. Numerical results are obtained for frames with various characteristics and support conditions when three illustrative examples from the literature are presented and discussed. The elastic buckling load is found to be strongly affected by semi-rigid joints and reveals that the proposed model is computationally very efficient with the expressions presented being general. The paper makes reference to the Eurocode 3 approach and those of other researchers in comparing the results. The proposed method is found to be more effective and simple to use, and yielding to very good results.

1. Introduction

Conventional analysis and design of steel frames assume either perfectly rigid or pinned joints. However, as is now well established, the real behaviour of the joints is between these two extreme cases: the most rigid joints always have some flexibility so that the joints are capable of transmitting a bending moment, whereas the pinned joints case always exhibit some rotational rigidity. In this intermediate case of semi rigid joints, some rotation with corresponding bending moments will develop between the beam and column elements. The concept of semi rigid joints in steel structures is well accepted [1–8]. Previous studies have indicated that in frame analysis, joint rotational behaviour must be considered. It is therefore necessary to incorporate the effect of joint flexibility in the frame analysis, otherwise the resulting internal forces and bending moments will contain errors [9–14].

Mathematical models were proposed in the past to fit the moment-rotation $(M-\theta)$ curves of joints, with various levels of complexity, using experimental data [1–4,9]. The response of the joint is dependent on the geometric and mechanical properties of its components. Because of the high number of the parameters influencing the behaviour of connections, accurate modeling of such behaviour becomes very complex. Globally, initial rigidity and the ultimate moment of the connection are the two most important [15].

Significant research has been carried out using mechanical models to study the joint's behaviour and to introduce their effect in the analysis of structures. Simões da Silva [12] proposed a generic model for steel joints under generalized loading. Ihaddoudène [16] presented a mechanical model of the connections, where the rigidity of the joint is represented by means of rotational and translational springs introducing the concept of non deformable element of nodes, thus describing relative displacements and rotations between the nodes and the elements of the structure. Eurocode 3 Part 1–8 refers [17], for the characterization of the joint mechanical response to the component method based on some different researches and amongst them Jaspart [10]. Nassani and Chikho [18] presented a formula to calculate the column ultimate load to simulate the behaviour of steel columns in sway structures. The structural benefits of using semi-rigid joints are widely recognized and there is nowadays a general agreement to include the

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Nomenclature

k_1, k_2: elastic constants of the springs in rotation at the nodes "i" and "j", respectively
D: denominator for the case where both the second order effects and rigidity of the joints are considered
D_1: denominator for the case where only the rigidity of joints is considered
w flexural rigidity per unit length $\frac{EI}{l}$
$\zeta_i(v)$, $\phi_i(v)$: functions including both axial forces and the rigidity of the joint for different situations

beam-column joint deformations in structural analysis. Various approaches are provided to include such an effect, for instance the finite element method [19,20]. The elastic stability of steel frames taking into account the effect of the joint flexibility and the elastic member instability are specific aspects to investigate.

Several authors [21–29] have presented models for determining the effective length factor of a beam-column with end restraints. Ermopoulos [21] presented a model for determining an equivalent buckling length of compression columns with semi rigid joints. Essa [22] proposed a design method for the evaluation of the effective length for columns in unbraced multistory frames. Raftoyiannis [23] presented the effects of the joint flexibility and elastic bracing system on the buckling load. Mageirou and Gantes [24], Gantes and Mageirou [25] proposed a model of an individual column representing a multistory frame where the member contributions converging at the bottom and top ends of the column are represented by equivalent springs. Xu and Liu [26] proposed a method for the stability analysis of semi braced steel frames with the effect of semi-rigid connections and the procedure of evaluating column effective length. Xu [27] presented a linear programming method to investigate stability strengths of unbraced steel frames subjected to variable loading, where the problem of determining the elastic buckling loads is expressed as a pair of maximization and minimization problems with stability constraints. A number of other alternative approximate effective length formulas are available in the literature; an overview is given in Hellesland [28] where it is shown how such formulas may be applied in system instability analysis of frames and comparisons with the exact effective length results have been carried out for isolated members. Cao et al. [29] presented a mechanical model of spring hinge ended column and design formulas to predicate the effective length factors were proposed.

2. Significance of the research

Chen et al. [30] proposed in an implicit form the stability functions derived from a slope-deflections approach. However, using the beam-column stiffness degradation approach and the stability functions, divergence occurs when the axial force of member is close to zero. A great deal of information on this subject have been presented by Chen et al. [30]. The proposed model however, is based on functions accounting for semi-rigid connections and predominant axial load, with an explicit formulation. Therefore, the formulation has the advantage of being explicit and simple to use, leading to very good results as is shown in the succeeding sections. Section 7 below gives a detailed description of the differences between the current approach and that proposed by Chen et al. [30].

3. Basic assumptions

A previous study carried out by Shayan et al. [31] has shown that the effects of the residual stresses and initial imperfections on the buckling load are of the order of 2% and less than 1%, respectively. Out-plane-effects were not considered as the study is only concerned with a two dimensional formulation of the problem. Furthermore, the axial load is applied through the centerline of the beam, and therefore no eccentricity is included in the analysis. Giraldo-Londono et al. [32] investigated the post-buckling and large deflections of beam-columns with non-linear semi-rigid connections, taking into consideration shear and axial effects. The authors obtained good results for the study of large-deflection and post-buckling behaviour of Timoshenko beam-columns with non-linear bending connections. Stamatopoulos [33] modeled a plane frame with the supports consisting of non-linear rotational and translational springs, employing an energy approach. The author obtained limit values for the rotational stiffness for which the flexible supports affect the buckling response of the frame.

Gorgun [34] presented a computer-based analytical method for geometrically nonlinear frames with semi-rigid beam-to-column connections, employing modified stability functions to model the effect of axial force on the stiffness of members. The linear and nonlinear analyses were applied for two planar steel structures. However, the stability functions are not specifically given in the model adopted. Nguyen and Kim [35] presented a numerical procedure based on the beam–column method for nonlinear elastic dynamic analysis of three-dimensional semi-rigid steel frames. Geometric nonlinearity is considered through the use of stability functions and geometric stiffness matrix. An independent hardening model is adopted to capture the dynamic behaviour of rotational. The authors used the SAP2000 software to verify the accuracy and efficiency of the proposed analysis through four numerical examples, but no validation against test results is presented.

MacRae et al. [36] have shown that in the elastic range, axial shortening may be safely ignored, and becomes more important once yielding in the members had occurred. As the current study is only concerned with investigating the elastic buckling, axial shortening is therefore ignored. Wongkaew and Chen [37] considered inelastic out of plane lateral torsional buckling in the advanced analysis for planar steel frame design. The authors showed that out-of-plane buckling is likely to govern the strength of non-sway frames and may control the design of some sway frames. As such, it is important that out-of-plane buckling is considered in advanced analysis, post-linear. However, in this linear elastic study, and for simplicity, lateral torsional buckling has not been considered, as the frames analyzed are assumed to be adequately restrained against the development of lateral torsional buckling failure, as is commonly the case in civil engineering structures.

Hence, the following assumptions were made in the development of the mathematical formulation of the model: (i) members are initially straight, piecewise prismatic; (ii) plane cross section remains plane after deformation; (iii) local buckling and lateral torsional buckling are not considered (since the problem is two-dimensional one); (iv) the panel zone deformation of the joint is neglected; (v) the effect of residual stresses on the system response (especially critical load) is ignored.