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Optimal design of Nonlinear Energy Sinks for SDOF structures subjected to white noise base excitations

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ABSTRACT

Nonlinear Energy Sinks (NESs) have recently received increasing attention from researchers because of their capability to passively absorb a significant amount of energy over a wide range of frequencies. In most studies, the dynamic response of the main structure coupled with one or more NESs is analysed for impulsive loading. In this paper, the performance of the NES attached to a Single Degree of Freedom (SDOF) system, under random Gaussian white noise base excitations, is investigated through several numerical simulations. In order to determine the optimal configuration for the device, four different objective functions are considered. Sensitivity analyses with respect to the intensity of the random loads, the mass ratio and the main parameters of the primary structure are presented. The authors propose an approximate design approach based on the use of the Statistical Linearization Technique, and an accurate empirical formulation linking the NES optimal parameters to the characteristic of the main structure and the random excitation. Numerical results are validated by Monte Carlo simulations. Finally, a numerical application for a 2-DOFs system equipped with a NES has been presented in order to investigate the applicability of the proposed empirical approach for Multi Degrees of Freedom structures.

1. Introduction

The growing urbanization and the constant advances in building technologies have led to increasingly tall, slender and lightweight structures, often vulnerable to dynamic loads (e.g. wind and earthquakes). In the last decades several control strategies [1] have been developed to protect tall structures against environmental actions, thus reducing structural vibrations. Dynamic vibration absorbers belong to the class of passive control systems. These devices have been widely used for civil engineering structures because of their capability to operate without requiring an external power source. This implies simplicity in design, maintenance and operation. However, conventional linear dynamic vibration absorbers are generally tuned to one of the modal frequencies of the main structure in order to suppress the contributions from fundamental modes. Consequently, they are very sensitive to the detuning, caused for example by creep, temperature effects or significant variation of the structural parameters.

The Nonlinear Energy Sink (NES) is able to resonate with any mode of the primary structure because of its nonlinear nature. It is, therefore, insensitive to frequency changes and more robust than linear Tuned Mass Dampers (TMDs) [2]. The NES consists of a small mass coupled to the main structure with a nonlinear spring and a damping element. It is capable to passively and irreversibly absorb and dissipate a significant amount of energy from the primary structure over a wide range of frequencies. This one-way irreversible transfer, known as passive target energy transfer, occurs in systems of damped oscillators mainly via transient resonance capture and has been widely investigated both analytically [3–8] and experimentally [9–14]. NES configurations can be categorized depending on the operating mechanisms, as grounded [3,4] or ungrounded [5,6], and as smooth [7,15,16] or non-smooth [9,10]. Specifically, a grounded configuration consists of an essentially nonlinear grounded oscillator coupled to the primary structure through a weakly linear stiffness, whereas in an ungrounded one the NES is directly and strongly coupled to the primary structure through the essentially nonlinear stiffness; a smooth NES configuration involves a cubic nonlinear stiffness, whereas a non-smooth one involves nonlinearities in the form of clearances or vibro-impacts. The performance of these types of NES has been numerically studied and experimentally tested under different loading conditions, such as impulsive [6,14,16,17], peri-

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odic [18–21], seismic [10,13,14,22] and stationary random loadings [2].

Despite the high capacity of this nonlinear absorber to reduce the dynamic response of the structures, the presence of the nonlinear stiffness term makes the NESs especially sensitive to loading perturbations. Therefore, the optimal design of the device parameters is of the utmost importance. Several optimisation criteria and objective functions have been introduced in literature, depending on the characteristics of the main system and the external excitations. In [23,24], a procedure to obtain optimal design of the energetic sinks with amplitude-phase variables has been proposed by considering the analytical solution of energy pumping problem for strongly nonhomogeneous two-Degree of Freedom (DOF) systems. A closed-form solution for the nonlinear stiffness of the NES bounds of stability has been proposed in reference [25], whereas an analytical tuning for the nonlinear stiffness of the NES under transient and harmonic excitations has been proposed in [26]. In [27], optimal design criteria to determine the minimum nonlinear stiffness required to activate the target energy transfer have been proposed for transient regime. A reliability-based design optimisation of the NES has been presented in reference [28] for the cases of two- and three- DOFs systems, under impulsive loading. Genetic Algorithms have been used to find the optimal parameters of NESs in order to mitigate the vibrations of beams subjected to moving loads in [29].

The performance of a NES coupled to a linear damped Eulero-Bernoulli beam has been also considered in [30], where an optimisation procedure has been performed for various boundary conditions with two different values of forcing amplitude. A two-DOFs structure, subjected to a band-limited white noise, has been investigated in [2]. In the latter study, the authors have shown that the optimal nonlinear stiffness decreases for increasing amplitude of the load. Moreover, the performance of the NES increases with the natural frequency of the main structure. However, these results have been obtained considering only one sample of white noise.

In this paper, the optimal parameters of a smooth ungrounded NES have been investigated for the case of a structure subjected to random base excitations, modelled as samples of a random Gaussian white noise process. The aim of this work is to analyse the dependence of the optimal NES parameters on the main structure characteristics and on the intensity of the random excitation in order to define a simplified optimisation procedure.

First, a simplified approach based on the Statistical Linearization Technique (SLT) has been used to obtain an approximate formula for the optimal nonlinear stiffness of the NES. The formula has been determined minimising the mean square error between the nonlinear system and the linearised one, subjected both to the same Gaussian excitation. The results have been compared with Monte Carlo simulations, in order to determine the error committed in evaluating the optimal parameters.

Then, a parametric analysis has been performed in order to investigate the dependence of the NES performance on the main characteristics of the system by varying the intensity of the random loading, the mass ratio between the NES and the primary structure, as well as the frequency and the damping ratio of the main structure. The numerical results are critically discussed, focusing on the sensitivity of the NES optimal nonlinear stiffness and linear damping. Empirical relationships among the optimal parameters, the amplitude of the load and the main system parameters are proposed for pre-design purposes and validated against the results of Monte Carlo simulations. Moreover, a simplified design of NESs for MDOF systems which involves the application of these empirical formulae is presented through a numerical application on a 2DOF model.

2. Structural model

A schematic diagram of the system considered herein is shown in Fig. 1. Known in literature as "Configuration II" [8] or "Type I" [22,31] NES, it consists of a SDOF primary structure connected to an ungrounded and lightweight NES through a pure cubic stiffness and a linear viscous damper. The equations of motion of the combined system can be written as following:

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_1x_1 + k_2(x_1 - x_2)^3 = -m_1\ddot{x}_g \\ m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1)^3 = -m_2\ddot{x}_g \end{cases} \quad (1)$$

in which m_1, c_1, k_1 and x_1 are mass, viscous damping coefficient, stiffness coefficient and displacement relative to the base of the main structure, respectively; m_2, c_2, k_2 and x_2 are mass, viscous damping coefficient, nonlinear stiffness coefficient and displacement relative to the base of the NES, respectively; dots mean derivative with respect to time, and $\ddot{x}_g$ is the base acceleration modelled as a zero-mean stationary Gaussian white noise, having Power Spectral Density (PSD) amplitude S_0 .

Dividing Eq. (1) by m_1, the nonlinear system of equations becomes:

$$\begin{cases} \ddot{x}_1 + \lambda_1\dot{x}_1 + \lambda_2(\dot{x}_1 - \dot{x}_2) + \omega_1^2x_1 + \kappa(x_1 - x_2)^3 = -\ddot{x}_g \\ \varepsilon\ddot{x}_2 + \lambda_2(\dot{x}_2 - \dot{x}_1) + \kappa(x_2 - x_1)^3 = -\varepsilon\ddot{x}_g \end{cases} \quad (2)$$

in which:

$$\lambda_1 = \frac{c_1}{m_1} = 2\zeta_1\omega_1; \quad \lambda_2 = \frac{c_2}{m_1}; \quad \omega_1^2 = \frac{k_1}{m_1}; \quad \kappa = \frac{k_2}{m_1}; \quad \varepsilon = \frac{m_2}{m_1}.$$

where ζ_1 and ω_1 are the damping ratio and the frequency of the primary structure and ε is the mass ratio between the NES and the main structure. Several papers showed that even simple systems may exhibit complex dynamics involving fundamental and subharmonic resonances, nonlinear beating phenomena and multifrequency responses [5,6,8].

An interesting feature of the NES is its ability to passively and irreversibly absorb and locally dissipate a significant amount of vibrational energy from the primary structures. Moreover, the NES is capable to interact over broad frequency bands, thus its performance is not significantly affected by structural frequency changes and it is more robust than linear passive absorbers. However, the NES performance is critically dependent on the amplitude of the external excitation since it was demonstrated that it only

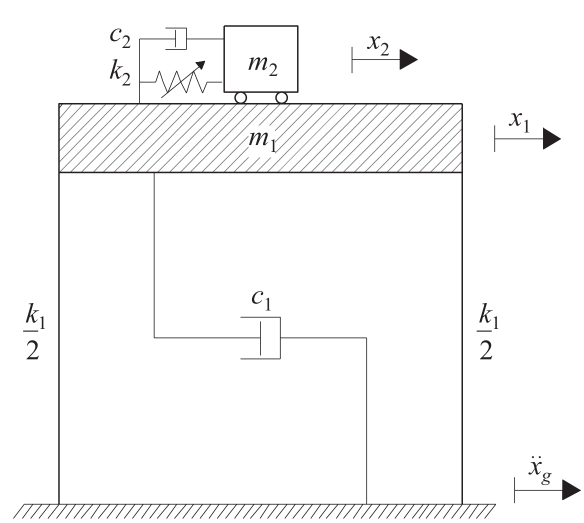

Fig. 1. SDOF structure coupled with a NES.