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A phononic band gap model for long bridges. The 'Brabau' bridge case

G. Carta [a,*], G.F. Giaccu [b], M. Brun [a]

[a] Dipartimento di Ingegneria Meccanica, Chimica e dei Materiali, Università di Cagliari, 09123 Cagliari, CA, Italy
[b] Dipartimento di Architettura, Design e Urbanistica, Università di Sassari, 07041 Alghero, SS, Italy

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ABSTRACT

In this paper, we study the dynamic flexural behaviour of a long bridge, modelled as an infinite periodic structure. The analysis is applied to the 'Brabau' bridge across the river Tirso in Italy. The approach reduces to a spectral problem leading to the analytical expression of the dispersion relation, which provides the ranges of frequencies for which waves do and do not propagate. The contributions of the bridge structural elements on the dispersive properties of the structure are investigated in detail. The direct link between frequency intervals determined by the proposed approach and distribution of eigenfrequencies of the full three-dimensional structure is demonstrated. The analysis of the unit cell allows to avoid the tedious computations required when using a finite element code, at least at a preliminary stage of the design. Finally, we demonstrate that a more precise prediction of the eigenfrequency ranges of the bridge can be obtained by studying a single repetitive cell numerically and imposing Floquet-Bloch conditions at its ends. The proposed approach can be implemented as a simple procedure to design structures with repetitive units, with the advantage of simplifying numerical simulations and reducing the computational cost.

1. Introduction

Long structures with repetitive units are very common in engineering applications for technological and economical reasons. In fact, in order to make the construction process faster and cheaper, many structures are made of precast elements that are connected *in situ* by appropriate joints. Typical repetitive structures are bridges and viaducts, pipelines and industrial warehouses.

'Repetitive structures' with discontinuities or inhomogeneities are characterised by bands of frequencies for which waves travel without attenuation ('propagation ranges') and intervals of frequencies for which waves decay exponentially in space ('non-propagation ranges'). In structures with an infinitely large number of units ('periodic structures'), these frequency bands are denoted as 'pass-bands' and 'stop-bands', respectively. The determination of the non-propagation ranges is of paramount importance in problems of vibration isolation. Indeed, if the repetitive structure is designed such that the frequency components of the external dynamic source (be it traffic, a vibrating machine or an earthquake) lie within the non-propagation ranges of the structure, the amplitudes of the waves generated by the source are significantly attenuated without the need to install dampers.

From the above considerations, we envisage that the availability of a simple modelling tool for the prediction of the non-propagation frequency intervals of a repetitive structure may be of high value for the engineering practice both at the design and verification stages. In particular, long bridges can be efficiently modelled as waveguides representing phononic band gap systems, and the associated dispersion relations enable to deduce the dynamic properties of the vibrating slender structures. A particular feature of the proposed model is the Floquet-Bloch study based on the analysis of a single cell, which drastically simplifies the analytical and numerical modelling of large scale structures and the post-processing analysis.

The first studies on waves in periodic systems date back to the 1950s [1–4]. Later, one-dimensional elastic structures – such as laminates, assemblies of rods and monodimensional lattices – were investigated by Mead [5], Faulkner and Hong [6], Martinsson and Movchan [7], Brun et al. [8], Carta and Brun [9], Brito-Santana et al. [10]. Lekner [11] examined the propagation of electromagnetic waves in stratified media. Wu et al. [12,13] used the spectral element method to study the band-gap properties of sandwich panels with corrugated cores and periodic Mindlin plate structures. Hull [14] studied laminated plates by means of a higher-order shear deformation model.

* Corresponding author at: Department of Maritime and Mechanical Engineering, Liverpool John Moores University, L3 3AF, Liverpool, UK.

E-mail addresses: giorgio_carta@unica.it (G. Carta), gf.giaccu@uniss.it (G.F. Giaccu), mbrun@unica.it (M. Brun).

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Dynamic properties of beam systems were analysed by Mead [15,16], Heckl [17], Romeo and Luongo [18], Xu et al. [19]. In this case, the governing equation of motion includes a fourth-order derivative with respect to the spatial coordinate, hence two waves (with or without amplitude attenuation) are expected to travel in each direction. By employing different techniques, Mead [20,21,15], Sen Gupta [22,23], Brun et al. [24] proved that the spectral analysis of Floquet-Bloch waves can be linked directly to the eigenfrequency intervals of a finite real structure. Carta et al. [25,26] examined the stop-bands and localisation phenomena produced by a diffuse damage on a long elastic two-dimensional strip; in addition, they showed that in the low- and medium-frequency regimes the limits of the stop-bands are predicted accurately by means of a lower-dimensional beam model with elastic junctions, the stiffnesses of which can be estimated via asymptotic techniques. A comprehensive analysis of the effects of different geometric distributions of structural elements having discrete and continuum nature is given in [27]. The effects of random perturbations in beam systems were investigated in [28–32]. Propagation of stable, non-linear waves in periodic buckled beams is discussed in [33,34], where a parametric resonance model shows that wave propagation depends on pre-compression and dynamic amplitude.

In this paper, we study the dynamics of long bridges with repetitive units. While Floquet-Bloch analysis on periodic structures is known in the literature, its applications in Structural Mechanics are far from being fully exploited, especially as a simple tool for the dynamic design of civil structures. We apply the quasi-periodic analysis to a particular case, namely the road and pedestrian 'Brabau' bridge across the river Tirso in Sardinia, Italy (see Fig. 1). This bridge is made of equally-long spans, consisting of five prestressed concrete beams supported by a dosseret standing on two pillars; the decks of adjacent spans are joined by the only slab in correspondence of the pillars. This construction technology, which ensures the achievement of span lengths up to 30 m, is widespread throughout the world due to its simplicity, speed of execution and relatively low cost. Moreover, the precasting phase guarantees durability and quality of the artifact.

We show that the vibration frequencies of the bridge are confined within specific ranges, which can be estimated through an analytical approach based on the dispersion analysis of the corresponding periodic structure. In particular, we focus our attention on the flexural motion of the bridge. As recommended by European Regulations [35], the vertical component of an earthquake has a relevant importance in case of prestressed concrete bridge decks and, thus, it has to be taken into account for an appropriate design of the structure.

In this work, we provide for the first time an analytical formula, given by Eq. (11), which can be used as a first approximation for the dynamic optimisation of the structure according to the specific demands imposed by the project. Eq. (11) embeds a class of structural parameters which strongly contribute to the dynamic behaviour of the bridge. We show the dynamic effects of these structural parameters and the limitations of the applied formula. Moreover, we provide a complementary simple numerical approach to overcome the limitations of the analytical formulation and to widen the range of applications of the quasi-periodic analysis to more complicated structural systems.

The paper is organised as follows. In Section 2 we describe the actual bridge and the structural models adopted to investigate its dynamic behaviour. In Section 3 we present the analytical method used to predict the eigenfrequency intervals of the bridge, and we discuss how these ranges are affected by the single contributions of the different structural elements. In Section 4 we compare the analytical results with the numerical values obtained from a finite element model, detailing the advantages of the analysis of the single unit cell. We conclude the paper with Section 5, where we provide some concluding remarks.

2. Description of the bridge

The bridge shown in Fig. 1 is located in the west coast of Sardinia, Italy. The structure, which was inaugurated in 2012, connects the city of Oristano with the touristic town of Torregrande.

2.1. Geometric and constitutive properties

The structure of the bridge is shown in Fig. 2. It consists of 41 spans of length $l \simeq 25.4$ m. The elevation of a typical span is 5 m (Fig. 2a). The deck is made of 5 prestressed concrete beams, which in the construction process are assembled by a special crane and subsequently connected to a superior slab, that is designed to resist the horizontal forces transmitted to the main structure by vehicular loads. All the spans are usually pre-designed as simply-supported beams, namely the cross-section and the steel reinforcement are usually calculated conservatively by assuming rigid pillars and pinned connections between the spans, even though in proximity of the pillars the slab generates a structural continuity which reduces the maximum bending moment in the mid-span cross-section. This structural continuity is not taken into account in the static design of the structure, but we will show in Section 4 that it affects significantly the dynamic behaviour of the bridge.

The height of the deck is approximately 1.91 m (Fig. 2b). Each beam, including the effective width of the slab (see Fig. 2c), has cross-sectional area $A_0 = 1.14\ \mathrm{m}^2$ and second moment of inertia $J_0 = 0.52\ \mathrm{m}^4$. The Young's modulus, Poisson's coefficient and mass density of the deck are given by $E = 34\ \mathrm{GPa}, \nu = 0.2$ and $\rho_0 = 2500\ \mathrm{kg/m}^3$, respectively. We point out that ρ_0 takes into account also the mass density of the reinforcement.

The deck is sustained by circular pillars of length $l_p \simeq 3.8$ m and diameter $d_p = 1.2$ m (Fig. 2a). The material properties of the pillars are $E_p = 31$ GPa$, \nu_p = 0.2$ and $\rho_p = 2500\ \mathrm{kg/m}^3$. There are two pil-

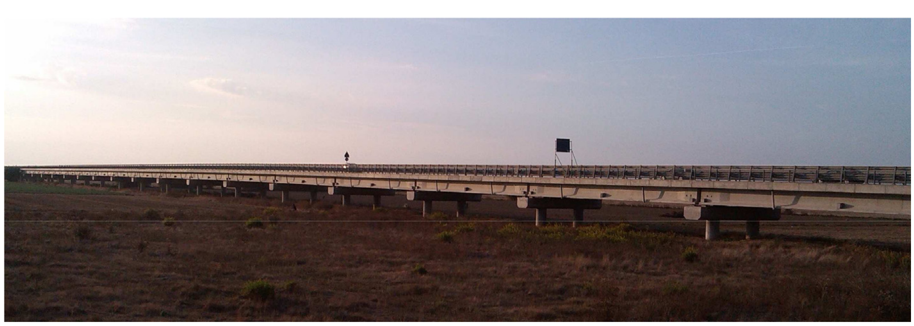

Fig. 1. The 'Brabau' bridge across the river Tirso in Oristano, Sardinia, Italy; 39°54′42.5″N 8°32′54.2″E.