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Stress at ultimate in unbonded tendons for ungrouted post-tensioned masonry beams

John M. García [a,*], Ricardo L. Bonett [a], Arturo E. Schultz [b], Christian Ledezma [c]

[a] Civil Engineering Department, Faculty of Engineering, University of Medellin, Medellin, Colombia
[b] Civil, Environmental and Geo- Engineering, Twin Cities, University of Minnesota, Minneapolis, USA
[c] Structural and Geotechnical Engineering Department, Escuela de Ingeniería, Pontificia Universidad Católica de Chile, Santiago, Chile

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ABSTRACT

Accurate estimation of tendon stress is crucial for calculating the flexural capacity of post-tensioned masonry members. Tendon stresses in bonded elements may be calculated based on strain-compatibility. For unbonded tendons, stresses depend on the relative displacement between the tendon's anchor points, and strain-compatibility is not totally applicable to calculate stresses. Masonry codes in some countries provide equations for unbonded, post-tensioned members that are based on modified strain-compatibility approaches for calculating stress increases in unbonded tendons at ultimate; some of these equations required calibration using statistical evaluation of experimental results and finite-element analysis. A new approach to calculate tendon stress increase, based on the theory of beam deformation, in the elastic zone, and a plastic hinge with a geometric curvature distribution in the inelastic region, is reported here for the calculation of the stress increase at ultimate. To compare the accuracy of code equations and that of the proposed methodology, a database of test results for post-tensioned, simply supported, flexure critical masonry beams has been used. This comparison shows that the proposed equation provides an accurate prediction of tendon stress at ultimate for post-tensioned masonry beams.

1. Introduction

A precise calculation of the flexural capacity of post-tensioned masonry elements has posed a great challenge over the years because it requires accurate estimates of the tendon stresses at ultimate. This task is relatively straightforward for post-tensioned members with bonded tendons due to strain compatibility between tendons and adjacent masonry, thereby enabling beam analysis using composite sections. However, strain-compatibility is not applicable to the tendon force increase in unbonded tendons: the stress increase in unbonded tendons depends strongly on the kinematic compatibility between masonry and tendons, as well as the deformation of the whole assembly [1]. Since the early 1950s, research on post-tensioned concrete beams has been conducted to determine the stress at ultimate in unbonded tendons. Some of the results of these investigations have been adopted by masonry design provisions such as those in Great Britain [2], Australia [3], the USA [4], New Zealand [5] and Canada [6].

The equations that are commonly used to calculate tendon stress in unbonded elements at ultimate are usually based on a modified strain-compatibility approach where equivalent plastic hinge lengths or strain (or bond) reduction factors are used. In this paper, a modified rational expression based on He and Liu's [7] methodology is extended to calculate the tendon stress at ultimate. The expression is based on the theory of beam deflection to calculate stress increase in the linear, elastic range. Additionally, the proposed expression in the nonlinear range, unlike He and Liu's [7] methodology that relies on a deflection reduction factor, is based on the concept of an equivalent plastic hinge that occurs at the section of maximum moment, near the mid-span of the beam, and utilizes an idealized distribution of curvatures in the beam. Moreover, the proposed equation in the elastic range, unlike the He and Liu's [7] methodology, relies on the use of the distance from extreme compression fiber to centroid of prestressing tendon, *d*, instead of tendon eccentricity, *e*, in order to avoid inconsistencies in structural elements with concentric tendons. In the inelastic stage, a very significant difference exists between proposed expression and He and Liu's [7] methodology, where instead of using a deflection reduction factor, it is based on the development of an equivalent plastic hinge in the region of the greatest bending

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* Corresponding author at: Carrera 87 #30-65 oficina 4-108, Medellín, Colombia.
E-mail addresses: jmgarcia@udem.edu.co (J.M. García), rbonett@udem.edu.co (R.L. Bonett), schul088@umn.edu (A.E. Schultz), ledezma@ing.puc.cl (C. Ledezma).

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moment, and an idealized nonlinear curvature distribution along the beam.

The proposed expression addresses loading type, initial tendon force, masonry compressive strength, beam length, tendon eccentricity, and tendon cross-sectional area. The proposed expression is validated by mean of physical data gathered from laboratory tests of full-scale tests post-tensioned hollow concrete masonry beams conducted as part of this investigation, as well as by data on post-tensioned solid clay masonry beams collected from the literature.

2. Review of analytical expressions and code equations to estimate tendon stress at ultimate

Over the past four decades, experimental and theoretical investigations have studied the influence of the principal factors that affect tendon stress increase in unbonded concrete members, and numerous design equations have been proposed with significant scatter in their prediction of experimental results [8]. The research on post-tensioned concrete members with unbonded tendons serves as a basis for studying post-tensioned masonry beams. From both theoretical and experimental research on post-tensioned concrete beams, the following basic parameters have been found to be the most influential: concrete strength, reinforcement amount, cross-section shape, span-to-depth ratio, and loading type [9].

The usual expression used to determine tendon stress in an unbonded tendon at ultimate ($f_{ps,u}$) adds the effective prestress after losses f_{se} and the subsequent stress increase at ultimate ($\Delta f_{ps,u}$) due to the external load applied after losses.

$$f_{ps,u} = f_{se} + \Delta f_{ps,u} \tag{1}$$

To estimate $\Delta f_{ps,u}$, Baker [10] introduced the strain or bond reduction method, which includes a bond reduction factor defined as the ratio between the strain change adjacent to the unbonded tendons and the strain change in the equivalent bonded tendons. Actual values for the bond reduction factor are influenced by loading type, tendon profile, and the relation between end and mid-span eccentricities [11]. Alternatively, an equivalent plastic hinge method has been proposed to calculate the tendon stress increase at failure [12]. This method considers the development of both elastic and inelastic zones along the beam, and its formulation is based on a mechanism comprising two equal length rigid bodies connected at the mid-span by a plastic hinge to represent the deformed shape of a simply supported beam at failure. The plastic hinge introduced at the maximum moment section redistributes any additional load to adjacent regions. The increase in tendon length between the anchorage end blocks is due to the plastic deformation in the plastic hinges.

The first design provisions for post-tensioned masonry were published by the British Standards Institution in 1985 [13], and the requirements were adapted from concrete codes used at that time. The use of strain compatibility between the tendon and the adjacent masonry makes the accuracy of code equations questionable [12,14,15]. In most codes, the expression to estimate the tendon force at ultimate utilizes the basic form of Eq. (2) and requires use of an equivalent rectangular stress distribution [Eq. (3)], as the masonry code provisions MSJC (2013) [4] suggest in section 9.3.2 (g).

$$f_{ps,u} = f_{se} + \frac{E_{ps}}{L}(d - c)\theta \tag{2}$$

$$c = \frac{a}{\beta_2} = \frac{f_{ps,u}A_{ps} + P_v}{\beta_1 \beta_2 f'_m b} \tag{3}$$

where c is the neutral axis depth; d is the distance from extreme compression fiber to centroid of prestressing tendon; L is the tendon length; E_{ps} is the modulus of elasticity of prestressing steel; a is the effective compression stress block depth; θ is the plastic hinge rotation; β_1 is the compression stress block magnitude factor; β_2 is the compression stress block depth factor; A_{ps} is the area of unbonded tendons; b is the width of beam section and P_v is the additional axial force. The principal difference among codes is the assumption regarding the behavior at ultimate, and the different empirical parameters obtained from test data and statistical correlations.

Some code assumptions have been investigated by Wight et al. [12], who observed that code expressions can lead to substantial errors for walls with low aspect ratios (defined as ratio between effective height and effective wall length, $h_e/l_e < 10$) and low axial force ratios (ratio between masonry stress and maximum masonry strength, $f_m/f'_m < 0.1$). Similarly, after reviewing the results of 54 laboratory tests on out-of-plane loaded post-tensioned masonry walls, Bean and Schultz [16], showed that the 2002 MSJC code [17] provided over-conservative estimates of tendon force increase, and proposed a modification for the tendon stress equations that establishes differences between restrained and unrestrained tendons, and these modifications were included in the 2005 MSJC code [18]. The formula found in the 2013 MSJC code [4] to calculate the tendon stress increase, is based on later research by Bean and Schultz [15], who indicated that the expression in the 2005 and 2008 MSJC codes was overly conservative with a large variance (COV > 0.75).

A summary of code equations is shown in Table 1, where some symbols have been modified from the original expressions to provide a standard notation and to avoid confusion. Although there are a large number of worldwide codes that regulate the design of masonry structures, this research has addressed the only codes that, to the researchers' knowledge, allow post-tensioned masonry.

The major differences between the code expressions concerns the values assumed for the variables such as β_1, β_2, E_{ps}, ε_{mu} and θ. However, the most important of these is the assumptions used for the calculation of plastic hinge rotation (θ). The British and Australian codes assume $\theta = \varepsilon_{mu}$, the United States code assumes $\theta = 0.03$, and the Canadian code assumes $\theta = 1/25 = 0.04$. It is worth noting that there is an order of magnitude difference between the plastic hinge rotation assumed in the British, Australian and New Zealand codes relative to that assumed in the US and Canadian codes.

3. Proposed methodology to estimate the tendon stress increase based on beam deformation

3.1. Tendon stress and beam deflection

A new methodology to calculate tendon stress increase based on basic beam deformation theory is proposed, where a nearly linear correlation between tendon stress and beam deflection exists during the entire loading regime. Total tendon elongation can be calculated assuming that: shear deformation is negligible, force along the tendon is constant, a plastic hinge is developed at ultimate, and friction between tendon restraints, surrounding masonry, and tendons is negligible.

The stress increase takes place between the time when "effective prestress" conditions are achieved (i.e., after all losses have taken place) and when additional vertical loading takes the beam to its ultimate limit state (i.e. when its nominal moment capacity is achieved). The increase arises from the relative deformation of the tendon between the anchorages, which, in turn, relates to the beam deflection. The initial conditions include a deflected tendon