Engineering Structures 140 (2017) 483–497

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

A flexure-shear Timoshenko fiber beam element based on softened damage-plasticity model

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ARTICLE INFO

Article history:
Received 21 September 2016
Revised 22 January 2017
Accepted 26 February 2017

Keywords:
Fiber element
Flexure-shear interaction
Timoshenko beam
Damage-plasticity model
Compression-softening
Mesh-objectivity

ABSTRACT

In this paper, a displacement-based fiber beam element including flexure-shear interaction is developed. The element is based on the conventional Timoshenko beam theory, and the section behavior is modeled with the well-known fiber section approach, where the section is divided into steel fibers and concrete fibers. The Menegotto-Pinto model is used for steel fibers and a multi-dimensional softened damage-plasticity model, which accounts for the compression-softening effect of reinforced concrete, is adopted for concrete fibers. The axial-flexure-shear interaction can be well reflected in both section level and material level since the normal-shear coupling is reflected naturally in the multi-dimensional material constitutive law. Besides, the concrete material parameters are related to fracture energy to avoid mesh-sensitivity issue of softening problems. The numerical implementation of the proposed element, including the finite element approximation and fiber state determination, is also discussed in detail. The element is validated through the test results for a series of simply supported reinforced concrete beams under monotonic loading and reinforced concrete columns and wall under cyclic loading, and the results indicate that the element is capable to reproduce the shear behaviors of reinforced concrete members under different loading cases.

1. Introduction

Beam-column element, or referred to as frame element, is widely used in nonlinear analysis of reinforced concrete (RC) frame structures due to its combination of numerical accuracy and computational efficiency. In the past two decades, several remarkable elements have been developed, among which the fiber beam element is most preferred in application [1–3]. The section is discretized with steel fibers and concrete fibers, and each fiber is assigned with a uniaxial material constitutive law. The axial-flexure interaction can be well reflected by the fiber section model, and the confinement effect of stirrups can also be considered. However, most fiber beam elements are settled in the framework of Euler-Bernoulli beam theory, where the shear deformations are neglected. This feature makes the elements not suitable for modeling RC members with obvious shear deformation, i.e., columns with shear span ratio lower than two [4].

In order to include the flexure-shear interaction in the beam-column elements, several approaches have been proposed. A detailed review can be found in [5]. The first way is to introduce flexure springs and/or shear springs at the end of the element such that the flexure-shear interaction can be obtained in the element level [6–11]. This method is simple and easy to be implemented, however, the element is actually a macroscopic phenomenological model and the parameters should be calibrated or identified according to the section properties (e.g., section area and reinforcing details) since the spring behavior is based on empirical force-deformation relations. The second way is to construct the element formulation based on the Timoshenko beam theory, and uses a multi-dimensional constitutive law for concrete fibers, thus the flexure-shear interaction can be obtained at both the material level and section level. Petrangeli et al. [12] first proposed a force-based model that accounts for shear effects using a constitutive model based on the micro-plane concrete model. Then the smeared-crack models are introduced into the Timoshenko beams, both displacement-based [4,13,14] and forced-based [15]. Some researchers also developed new Timoshenko beams based on damage mechanics [16]. All of such elements seem to be more complicated compared with those in the first way, however, they are physical meaningful and more accurate, thus can be generally used in engineering practice. Moreover, the detailed stress and strain

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http://dx.doi.org/10.1016/j.engstruct.2017.02.066
0141-0296/

states of section fibers can be achieved. Obviously, it is the tendency in structural engineering to combine Timoshenko fiber beam with multi-dimensional material constitutive laws to reflect flexure-shear interaction with the development of numerical methods and computer technology.

Although several models can be used for concrete constitutive law, the smeared-crack models, i.e., the modified compression field theory (MCFT) family [17,18] and the softened truss model (STM) family [19–22], and the damage-plasticity models [23–26] seem to be the favorite ones. The MCFT and STM rely on the experimental studies of RC panels subjected to shear, and establish the constitutive model for cracked reinforced concrete in terms of average stress and average strain. The two typical characteristics of reinforced concrete subjected to shear, i.e., tension-stiffening and compression-softening, can be represented by the MCFT and STM. The models are the pioneer work in determining the realistic shear behavior of RC structures, and their main contribution is the introduction of softening coefficient that considers compression-softening. However, these models are based on the empirical uniaxial stress-strain relationships of concrete, whose hysteretic rules are rather complex, leading to some numerical issues from the computational point of view, especially for cyclic loading and dynamic loading [27,28].

On the other hand, damage mechanics offers an alternative way to construct the concrete constitutive model. The degradation of material is represented by a generalized internal variable, i.e., damage, and the residual deformations are explained by the plastic strains. Based on the thermodynamics, damage and plasticity of concrete are organized in a unified framework. Damage-plasticity models have a solid theoretical foundation, ensuring that they can be used as a standard tool for nonlinear analysis of RC structures [27–29]. Unfortunately, the compression-softening effect of reinforced concrete in shear has not been considered in damage models up till now, causing an overestimate of the shear behavior of RC structures.

When using local constitutive models in beam elements for nonlinear analysis of RC members, the localization issues may arise for strain-softening problems. The plastic deformations will be concentrated on a single element (for displacement-based element) or a single integration point (force-based element) [30]. Several regularization methods have been developed for both kinds of elements, i.e., constant fracture energy methods [30], nonlocal integral models [31,32], gradient models [33], and plastic hinge integration methods [3,34,35]. However, most approaches have been applied to Euler-Bernoulli beams only. The localization issues for Timoshenko beams, especially the flexure-shear fiber beam, have rarely been discussed [5].

Based on the above-mentioned background, this paper aims at developing a displacement-based Timoshenko fiber beam element that including flexure-shear interaction. A softened damage-plasticity model, in which the softening coefficient in MCFT and STM is adopted to modify the compressive damage variable to reflect compression-softening, is used for concrete. In addition, the model parameters is related to the fracture energy to avoid mesh-sensitivity in strain-softening problems. The paper is structured as follows: in Section 2, the softened damage-plasticity model is first introduced. Section 3 gives the formulation and finite approximation of the classical Timoshenko beam element. The fiber section state determination is discussed in Section 4. In Section 5, a series of RC beams under monotonic loading and RC columns and wall under cyclic loading are simulated by the proposed element and Euler-Bernoulli element, demonstrating the performance and advantage of the proposed element. Finally, the conclusions are drawn in Section 6.

2. Softened damage-plasticity model for concrete

2.1. Theoretical framework

The bi-scalar damage-plasticity model developed by Wu et al. [26] is adopted as the framework in the present paper. According to the damage-plasticity theory, the strain tensor $\boldsymbol{\epsilon}$ is split to a elastic part and plastic part

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^e + \boldsymbol{\epsilon}^p \tag{1}$$

where the superscripts "e" and "p" denote the elastic part and plastic part, respectively.

Calling for the hypothesis of strain equivalence [23], i.e., the strain undergoing a damaged state in the Cauchy stress space is equivalent to the undamaged state in the effective stress space, the effective stress $\overline{\boldsymbol{\sigma}}$ of concrete can be expressed as

$$\overline{\boldsymbol{\sigma}} = \mathbb{E}_0 : \boldsymbol{\epsilon}^e = \mathbb{E}_0 : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^p) \tag{2}$$

where $\mathbb{E}_0$ is the fourth-order initial elastic stiffness tensor.

Considering the different behaviors of concrete material under tension and compression, the effective stress can be further decomposed into a positive (tensile) part and a negative (compressive) part, i.e.,

$$\overline{\boldsymbol{\sigma}} = \overline{\boldsymbol{\sigma}}^+ + \overline{\boldsymbol{\sigma}}^- \tag{3}$$

where the superscripts "+" and "−" denote the positive part and negative part, respectively. The spectral decomposition method [26] can be followed for Eq. (3) to obtain the positive and negative effective stresses

$$\begin{cases} \overline{\boldsymbol{\sigma}}^+ = \mathbb{P}^+ : \overline{\boldsymbol{\sigma}} \\ \overline{\boldsymbol{\sigma}}^- = \mathbb{P}^- : \overline{\boldsymbol{\sigma}} \end{cases} \tag{4}$$

where $\mathbb{P}^{\pm}$ are the projective tensors, which define the eigen-tensors of the fourth order damage tensor, and are expressed by the principal directions of effective stress [25,26]

$$\begin{cases} \mathbb{P}^+ = \sum_i H(\hat{\overline{\sigma}}_i) \boldsymbol{p}_i \otimes \boldsymbol{p}_i \otimes \boldsymbol{p}_i \otimes \boldsymbol{p}_i \\ \mathbb{P}^- = \mathbb{I} - \mathbb{P}^+ \end{cases} \tag{5}$$

in which $\hat{\overline{\sigma}}_i$ is the i-th principal stress of the effective stress and $\boldsymbol{p}_i$ is the associated principal direction; $H(\cdot)$ is the Heaviside function; $\mathbb{I}$ is the identity tensor.

In order to establish the constitutive law through a thermodynamic manner, the Helmholtz free energy (HFE) potential should be defined. Meanwhile, introducing two damage variables d^+ and d^- to represent the tensile and compressive damages respectively, the HFE ψ can be expressed as

$$\psi = (1 - d^+)\psi_0^+ + (1 - d^-)\psi_0^- \tag{6}$$

where damage $d^{\pm}$ are the continuum measures of material degradation induced by cracks and defects in the sub-scale; $\psi_0^{\pm}$ are the initial HFE, which can be also split to a elastic part $\psi_0^{e\pm}$ and plastic part $\psi_0^{p\pm}$, and the expressions for each part are

$$\begin{cases} \psi_0^{e\pm} = \frac{1}{2} \overline{\boldsymbol{\sigma}}^{\pm} : \boldsymbol{\epsilon}^e \\ \psi_0^{p\pm} = \int \overline{\boldsymbol{\sigma}}^{\pm} : d\boldsymbol{\epsilon}^p \end{cases} \tag{7}$$

According to the second principle of thermodynamics, the Clausius-Duhem inequality can be expressed as

$$\dot{\gamma} = -\dot{\psi} + \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}} \geqslant 0 \tag{8}$$

Substitution of Eqs.(6) and (7) in Eq. (8) leads to the final expression of the constitutive relation, i.e.,