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Analytical study on reinforced concrete frames subject to compressive arch action

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ABSTRACT

Reinforced concrete beams can resist applied vertical loads through the development of compressive arch action (CAA) when adequate horizontal restraints are provided at the ends. However, at the frame level, CAA tends to push adjoining columns outwards and may induce premature flexural or shear failure to the columns. Therefore, under CAA, flexural and shear resistances of the connecting columns need to be examined. This paper describes an analytical study on the behaviour of reinforced concrete frames subject to CAA. In the study, lateral and rotational stiffness of reinforced concrete columns is determined based on a rigid-plastic assumption. CAA of reinforced concrete frames is investigated through an analytical model, in which deformations of columns are taken into consideration. Besides, shear force and bending moment in the column are calculated from equilibrium to shed light on the possible failure mode of columns under CAA. Parametric studies are also conducted to investigate the dominant parameters on the behaviour of columns. Finally, recommendations are provided for the design of reinforced concrete columns against CAA in the connecting beam.

1. Introduction

When restrained against horizontal movements, reinforced concrete beams and slabs can develop significant net axial compression force and arch action between compression zones, which in turn enhances the load resistance of beams and slabs. Park proposed a rigid-plastic model for estimating the compressive arch action (CAA) of reinforced concrete slabs [1]. It was assumed in the model that tensile reinforcement has yielded and compressive concrete attains its ultimate strain. Besides, flexural deformations of the beam are concentrated at the plastic hinges at the beam ends, whereas the beam segment itself remains rigid. Thereafter, shortening of the beam induced by axial compression force was introduced to the model [2,3]. The effects of imperfect boundary conditions, such as rotation of supports and connection gaps in horizontal restraints, were also incorporated in the model by Guice et al. [4] and Yu and Tan [5]. The rigid-plastic model was further extended to the elastic stage of the deformed beam by determining the strain profile across the beam end sections according to compatibility [6], in which strain-hardening behaviour of high-performance fibre-reinforced concrete was taken into consideration as well. To date, the effects of axial and rotational restraints, beam span-depth ratio and reinforcement ratio on the CAA of beams have been well investigated [5]. Furthermore, pseudo-static resistance of beams was calculated based on the energy-balance method proposed by Izzudin et al. [7]. It was concluded that development of CAA increases the quasi-static load capacity but reduces the ductility of beams, thereby enhancing the pseudo-static resistance marginally [6].

However, focus of previous analytical studies was primarily placed on the CAA of reinforced concrete beams with fairly rigid boundary conditions, in which stocky column stubs with sufficient flexural and shear resistances were provided at the ends of beams [8–13]. Limited effort was made to evaluate the behaviour of comparatively slender columns when subject to CAA in the adjoining beams. Experimental results of reinforced concrete frames demonstrate that columns may exhibit shear failure as a result of additional shear force in the beam-column joints [14–16]. Therefore, it is imperative to assess the resistances of side columns and beam-column joints under CAA in the connecting beams.

This paper presents an analytical study on the behaviour of reinforced concrete frames with realistic columns adjoining to bridging beams. An analytical model for CAA of reinforced concrete frames is introduced. In the model, the compatibility condition for bridging beams developed by Yu and Tan [5] is adopted in the study. To determine the strains and forces in steel reinforcement

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and concrete, the method proposed by Kang and Tan [6] is utilised, and lateral and rotational stiffness of columns is considered. A rigid-plastic model is also derived to determine the stiffness of reinforced concrete columns. The analytical model for CAA is validated against experimental results of Kang and Tan [14,17], Yu [18] and Sadek et al. [15] and reasonably good agreement is achieved. Finally, parametric studies are conducted to investigate the dominant parameters on potential flexural and shear failure of columns. Special attention is paid to the shear force in beam-column joints and bending moment acting on columns, endeavouring to gain insight into the potential failure mode of columns subject to CAA in adjacent beams.

2. Analytical model for compressive arch action

For an exterior reinforced concrete frame, which comprises of a bridging beam, a middle joint and two side columns and is simply-supported at the inflection points of the columns, development of CAA is accompanied by net axial compression force in the beam. Due to insufficient lateral and rotational stiffness of the columns, side beam-column joints are pushed outwards by a horizontal displacement of t as a result of beam axial compression force. Meanwhile, hogging moment at the side joint face generates a rigid-body rotation of Θ in the side joints. Fig. 1 shows the deformed geometry of the frame. Thus, in deriving the analytical model for CAA, deflection of the column and rotation of the joint need to be considered.

2.1. Compatibility condition

Due to symmetry, the single-span beam, middle and side joints are isolated from the prototype frame. Fig. 2 shows the compatibility condition of the beam. The horizontal distance between points A and B can be determined from the movement and rotation of the support as well as the deformed geometry of the beam, as expressed in Eq. (1). Accordingly, vertical displacement δ at the middle joint can be calculated from Eq. (2).

$$l + 0.5\varepsilon_b h_c + t - 0.5h\tan\Theta = (l(1-\varepsilon_b) + (h-c_1)\tan(\varphi-\Theta) - c_2\tan\varphi)\cos\varphi \quad (1)$$

$$\delta = (l + 0.5\varepsilon_b h_c + t - 0.5h\tan\Theta)\tan\varphi \quad (2)$$

where l is the clear span of the beam; ε_b is the axial compressive strain of the beam; h_c and h are the depths of the column and the beam, respectively; t is the horizontal movement of the side joint caused by beam axial force; c_1 and c_2 are the neutral axis depths at And 12; φ is the rotation of the beam; Θ is the rotation of the side joint; and δ is the vertical displacement of the middle joint.

Besides the whole beam, compatibility at Sections 1 and 2 has also to be satisfied. It is assumed that the strain of top longitudinal

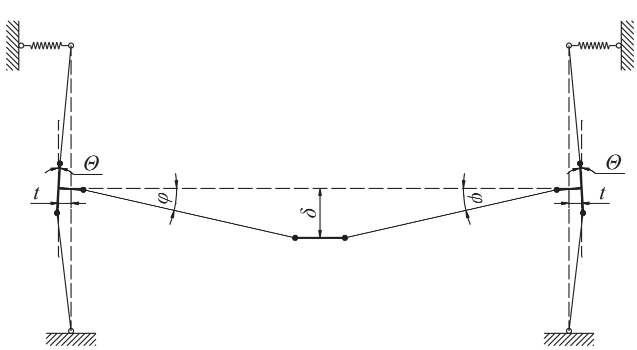

Fig. 1. Deformed geometry of a reinforced concrete frame.

reinforcement is zero at the inflection point of the beam and varies linearly along the beam length, as shown in Fig. 2. Correspondingly, deformations of top reinforcement at Sections 1 and 2 can be determined from the summation of steel strains between the inflection point and end sections, as expressed in Eqs. (3) and (4). Lengths of beam segments l_1 and l_2 can be quantified from bending moments at Sections 1 and 2.

$$\Delta l_1 = (h - c_1 - a_{s1})\tan(\varphi - \Theta) = 0.5\varepsilon_{s1} l_1 \quad (3)$$

$$\Delta l_2 = (c_2 - a'_{s2})\tan\varphi = 0.5\varepsilon'_{s2} l_2 \quad (4)$$

where a_{s1} is the spacing between the extreme tension fibre of the beam and the centroid of tensile reinforcement at Section 1; a'_{s2} is the spacing between the extreme compression fibre of the beam and the centroid of compressive reinforcement at Section 2; ε_{s1} and ε'_{s2} are the respective strains of reinforcement at Sections 1 and 2; and l_1 and l_2 are the distances between the inflection point of the beam and Sections 1 and 2, respectively.

2.2. Equilibrium condition

Besides compatibility, equilibrium of the deformed beam can be established from the free-body diagram, as shown in Fig. 3. At a vertical displacement δ of the middle joint, vertical load P on the middle joint is equilibrated by bending moments M_1 and M_2, beam compression force N and self-weight q of the beam. Eq. (5) expresses the equilibrium of the beam. Net axial compression force in the beam is correlated to its axial strain via Eq. (6).

$$P = \frac{2(M_1 + M_2 - N\delta - ql^2/2)}{l} \quad (5)$$

$$N = bhE_c\varepsilon_b \quad (6)$$

where M_1 and M_2 are the bending moments at Sections 1 and 2, calculated from internal forces (see Fig. 2); N is the beam axial compression force; q is the self-weight of the beam; P is the vertical load on the middle joint; b is the beam width; and E_c is the elastic modulus of concrete.

2.3. Constitutive models for concrete and steel reinforcement

To determine the forces sustained by concrete and steel reinforcement, constitutive models for concrete and reinforcing bars have to be defined. Previous analytical studies assumed that extreme compression concrete fibre attains its ultimate strain and equivalent rectangular concrete stress block was used to calculate the compression force [1,5,19]. However, it leads to substantial overestimation of vertical load before the CAA capacity is attained. To accurately predict the ascending branch of the vertical load, stress-strain model for concrete proposed by Mander et al. [20] is utilised in the analytical model instead of equivalent compressive stress block. Compression force in concrete is calculated through integration of compressive stress across the compression zone. Similar approach is also used to compute the contribution of concrete compression force to bending moment. Besides, a bilinear stress-strain relationship is employed for tensile steel reinforcement. For compressive reinforcing bars, a linear unloading branch is defined, with its stiffness identical to the elastic modulus of steel reinforcement.

2.4. Solution procedures

Prior to analysis, material and geometric properties and boundary conditions of reinforced concrete frames need to be determined. With compatibility, equilibrium and constitutive models