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Yield-line plasticity and tensile membrane action in lightly-reinforced rectangular concrete slabs

Ian Burgess

Department of Civil and Structural Engineering, University of Sheffield, Sheffield S1 3JD, United Kingdom

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ABSTRACT

The paper provides the systematic derivation of a new analytical approach to tensile membrane action of lightly-reinforced thin concrete slabs at large deflections. The basic motivation for the work comes from the recent use of tensile membrane action as an enhancement, in the fire condition, of the capacity of the thin concrete slabs which are normally made composite with downstand steel beams, at temperatures which have substantially degraded the contribution of these steel beams. The method accepts as a premise that such slabs form a pattern of localized yield lines as an initial small-deflection failure mechanism, and that these yield lines retain their positions as subsequent deflection occurs. As the slab deflects, maintaining the correct kinematics of the articulation and displacement of the system of slab facets, interacting across the yield lines, is extremely important to the horizontal equilibrium of the slab. In this process it becomes necessary to re-think the basic assumption of traditional yield-line theory that any local cross-section of unit width along a yield line equilibrates the force of its concrete compression block with the yielded steel's tension force, producing constant plastic moment capacities for the mesh in either direction along any yield line. In the approach set out in this paper only overall equilibrium of the system of facets needs to be maintained. As in normal rigid-plastic analysis, concrete acts only when compressed, and then at its compressive strength, and steel acts at its tensile yield strength whilst it remains intact. However, steel in either direction can fracture when the local crack-width causes its local strain to exceed its fracture ductility. When the rebar crossing the diagonal yield lines begins to fracture this generally indicates that the slab's capacity is about to reduce with further deflection. The paper does not attempt to address how a rebar's free length across a discrete crack is generated, or the limiting crack widths implied, but this is shown in a range of examples to be a major issue if tensile membrane action is to be used in practice to enhance the capacity of slabs, for example in hazard loading situations. It is important that principles be established in future to quantify this aspect of rebar ductility.

1. Introduction

The strength behaviour of concrete slabs, as well as the investigation of calculation methods suitable for routine use in design, generated significant interest during the 1950s and 1960s, following the initial publication in Swedish of yield-line theory by Johansen [1] in 1943; this was later translated into English, but had in any case rapidly been picked-up, explained and developed by others [2,3]. Johansen's theory concerns the plastic limit-states of reinforced concrete slabs, in principle of any shape, size, support conditions and reinforcement ratio, although in practice being most relevant to under-reinforced slabs. It is postulated that, after undergoing some small elastic displacement the slab develops a pattern of discrete plastic hinges which, when complete, comprise its "failure" mechanism. The development sequence for a simply supported rectangular slab is illustrated in Fig. 1.

The slab forms flat facets between the linear plastic hinges, known as yield lines, at which relative rotations take place. The load capacity of a given mechanism geometry can be calculated simply by equating the loss of potential of the external loading in deflecting the slab's surface in this way with the plastic work done along the yield lines in articulating to create the deflection. For any assumed mechanism the resulting load capacity prediction lies above or equal to the real limiting capacity; in the case of the rectangular slab shown in Fig. 1 the predicted load capacity has to be minimized with respect to the coordinate nl of the intersection of the central and diagonal yield lines in order to achieve the exact limiting value. A floor slab may be continuous across many individual edge-supported panels, in which case additional yield lines must be considered just inside the continuous edges.

E-mail address: ian.burgess@sheffield.ac.uk

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Nomenclature

BRE/Bailey method [13,14,22,23]

e	weighted-mean enhancement factor
$e_1,\ e_2$	enhancement factors for triangular and trapezoidal slab facets
n	dimensionless coordinate of yield line intersection as a proportion of L (long span)
α	slab aspect ratio (x:y)
μ	orthotropy ratio, equivalent to Ω in current method (below)

Current method

$A_1,\ A_{2y}$	concrete stress block areas on diagonal and central yield lines
C	concrete resultant force across a diagonal yield line
$C_{x2},\ C_{y2}$	concrete x- or y- direction resultant forces on central yield line
$f_{px},\ f_{py}$	steel strengths per unit width in x and y directions
f_c	concrete strength
l	y-dimension of slab
n_x, n_y	dimensionless coordinates of yield line intersection in x- and y-aligned mechanisms
p	distributed load intensity on slab
r	aspect ratio of slab (x:y)
S	resultant shear force along a diagonal yield line
t	thickness of slab
T_{x1}, T_{x2}	tensile resultant forces in x-aligned mesh
$T_{y1},\ T_{y2}$	tensile resultant forces in y-aligned mesh
u, v	movements of a point on a crack-face in x and y directions
$u_C,\ v_C$	movements of centroid of diagonal concrete stress block in x and y directions
$u_{T_{x1}},\ v_{T_{y1}},\ v_{T_{y2}}$	movements of resultant mesh forces on diagonal and central yield lines
$v_{C_{y2}}$	movement of centroid of central concrete stress block in y direction
W_e	external work: loss of potential of load on slab
W_i	internal work: plastic movements of steel and concrete
x, y, z	coordinate system
$x_{CA,1},\ y_{CA,1},\ z_{CA,1}$	coordinates of concrete stress-block centroids on diagonal and central yield lines
$x_{lim,1y}$	limiting x coordinate of unbroken y-direction reinforcement
$x_{t,1}$	x coordinate at which y reinforcement emerges from compressive stress block
$y_{lim,1x}$	limiting y coordinate of unbroken x-direction reinforcement
$y_{t,1}$	y coordinate at which x reinforcement emerges from compressive stress block
$z_1,\ z_2$	depths of concrete stress block at slab corner and yield line intersection
γ	angle of diagonal yield line to y axis
δ_A	deflection of centre of slab
$\Delta_x,\ \Delta_y$	x and y movements of facets at corner of slab
$\Delta_{lim,x},\ \Delta_{lim,y}$	limiting x and y crack widths at which reinforcement fractures
$\eta_x,\ \eta_y$	dimensionless limiting crack widths $\eta_x = \frac{\Delta_{lim,x}}{l}$ and $\eta_y = \frac{\Delta_{lim,y}}{l}$
θ	rotation of slab Facet 1 about x axis
ϕ	rotation of slab Facet 2 about y axis
μ	mesh depth as a proportion of slab thickness
$\lambda_x,\ \lambda_y$	dimensionless strength ratios $\lambda_x = \frac{f_{px}}{f_c l}$ and $\lambda_y = \frac{f_{py}}{f_c l}$
$\psi_1,\ \psi_2$	dimensionless stress block depths $\psi_1 = z_1/l,\ \psi_2 = z_2/l$
τ	dimensionless slab thickness t/l
Ω	orthotropy factor f_{py}/f_{px}

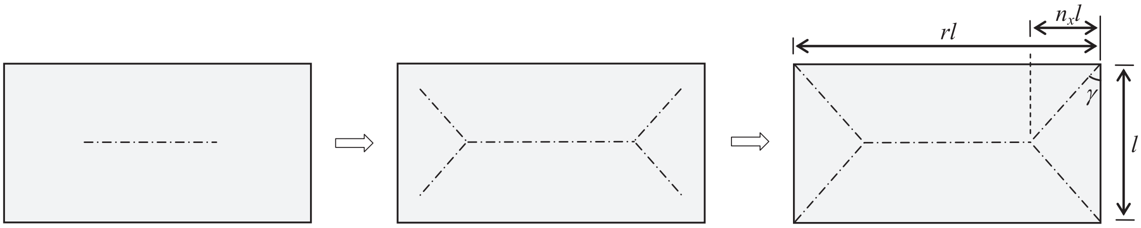

Fig. 1. Stages of development of a yield-line mechanism in a concrete slab.

Tests by Ockleston [4] in the 1950s, on an existing reinforced concrete building, showed load capacities of slabs considerably in excess of those predicted by either yield-line theory or by the simpler Hillerborg [5–7] strip rationalisation. This was later confirmed [8–10] in many academic research tests. In conventional two-way-spanning reinforced concrete slabs, with flat soffit and moderate span as well as restraint to horizontal edge movement given by adjacent slab panels, the slab depth is usually sufficient for a compressive membrane action (CMA) to account for this apparent strengthening. This effect is actually an arching action which creates a shallow dome-shaped surface of resultant compressive thrust within the thickness of the slab. In slab panels which are thinner relative to their overall dimensions this mechanism may initially occur if the necessary horizontal edge-restraint is present, but will undergo a "snap-through" instability at very low deflection, effectively inverting the thrust surface which then acts as a hydrostatic tensile membrane field. In contrast to compressive membrane action, this tension can be equilibrated internally within the slab panel by a narrow circumferential field of principal compression stress; this is facilitated by concrete's strength in compression. This mechanism is known as tensile membrane action (TMA), which demands only transverse support around all the slab's edges in order to make it work.

The load-carrying mechanism in the horizontal plane, illustrated in Fig. 2, is analogous to the force transfer in a bicycle wheel (but without the pre-tension in the spokes), with a radial tension field balanced by circumferential compression. In the transverse direction the external loading within any closed area is largely supported by integration of the vertical components of the same radial tension, at their corresponding inclinations, around its perimeter. In Fig. 2 the plan view of a rectangular slab shows simple vertical support provided around its edges.