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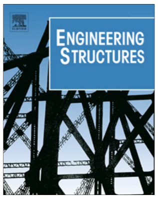

Analytical modeling of reinforced concrete columns subjected to bidirectional shear

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ABSTRACT

Under general seismic loading, reinforced concrete columns may be subjected to lateral loads in more than one direction. Available experimental data on columns subjected to bidirectional forces indicate that higher levels of damage and a higher loss of ductility and strength have been observed compared to similar tests under unidirectional shear forces. In this study, an experimental program was conducted in which six lightly reinforced concrete columns were subjected to unidirectional and bidirectional cyclic shear forces. This observation was used to identify the mechanisms and parameters governing the behavior of columns subjected to cyclic bidirectional lateral loads. Hence, a new conceptual model was developed to obtain the capacity of member. The shear forces were analyzed and an analytical formulation was derived to account for the effects in the concrete stress-strain relationship, the moment-curvature diagram and the plastic hinge length. These equations were used along with a structural model with concentrated plastic hinges to obtain the capacity curve of the column. The results of the formulations developed were verified using the results of the experiments performed on columns subjected to unidirectional and bidirectional cyclic lateral forces.

1. Introduction

Under seismic action, reinforced concrete columns can be subjected to lateral displacements in several directions. The magnitude of the displacement in each direction depends on the dynamic response of the structure, the orientation of the structure in the earthquake direction, the distance to the epicenter and the magnitude and direction of the aftershocks, among other factors. Such movements produce cyclic flexural and normal forces in combination with bidirectional shear forces.

In the limited available experimental data on columns subjected to bidirectional forces, higher levels of damage and a higher loss of ductility and strength have been observed compared to similar tests under unidirectional shear forces. These experimental data are compiled in [1] and subsequently tests were conducted by [2–5]. In the scientific literature, only a decrease in the contribution of concrete to the shear strength was mentioned in [6] and was later refined in [7,8]. It is essential to elucidate this mechanism to make recommendations and develop methodologies for the assessment of columns under bidirectional seismic forces.

To gain insight into the mechanism of the aforementioned problem, an experimental program was carried out on circular columns with light transverse reinforcement, common in older existing reinforced concrete buildings. Those were affected by shear forces under the action of unidirectional and bidirectional lateral loads [9]. The primary results of the study were that the strains in the stirrups produced by the shear forces and the confinement action could be accumulated by alternating the direction of the lateral loads.

The primary objective of this study was to identify the mechanisms and parameters governing the behavior of columns subjected to bidirectional lateral loads. For this purpose, the effects of the shear forces were analyzed at the material, section and member levels, and analytical expressions were derived to account for these effects in the concrete stress-strain relationship, the moment-curvature diagram and the plastic hinge length. These equations were used along with a structural model with concentrated plastic hinges to obtain the capacity curve of the column. The results of the formulations developed were verified using results from the experiments performed on columns subjected to unidirectional and bidirectional cyclic lateral forces. Good agreement was obtained between the experimental observations and the formulation developed.

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Notation

A_{st}	is the area of the stirrups or the spiral
D_c	is the center-to-center diameter of the stirrups
E_c	is the concrete modulus of elasticity
E_s	is the elastic modulus of the longitudinal reinforcing steel
E_{st}	is the elastic modulus of the transverse reinforcement
f_c	is the unconfined concrete compression strength
f_{cc}	is the peak strength of the confined concrete
f_y	is the yielding strength of the longitudinal reinforcing steel
f_{yt}	is the yielding strength of the stirrups
L	is the shear span
L_e	is the column length plus the length corresponding to the strain penetration
L_p	is the equivalent plastic hinge length
M_f	is the ultimate bending moment capacity
M_{fv}	is the effective flexural strength
M_y	is the yielding moment
M_u	is the ultimate moment
P	is the axial load
s	is the stirrups spacing
V_d	is the shear force demand
V_s	is the transverse reinforcement contribution to shear strength
$V_s^{\theta=45^\circ}$	is the shear force strength provided by the stirrups for a crack inclination angle of 45°
V_p	is the arch effect contribution to shear force strength
V_u	is ultimate shear force strength
x	is the distance required to develop
z	is the lever arm
α	is a factor that represents the confinement efficiency
α_b	is the coefficient of the strain penetration

Δ_f	is the flexural displacement
Δ_p	is the plastic displacement
Δ_s	is the displacement from the shear
Δ_y	is the elastic displacement
Δ_θ	is the displacement from strain penetration of the longitudinal rebar
ΔM_v	is the increment in the internal bending moment at the section that is produced by the shear force
ΔT	is the increment in the tensile force at the longitudinal reinforcement from shear
ε_{cp}	is the ultimate strain in the unconfined concrete
$\varepsilon_{cu,e}^i$	is the effective deformation capacity of the concrete
ε_{sv}^j	is the strain induced by the shear forces in j=y or x direction
ε_{sc}^i	is the strain produced by the dilatancy of concrete in compression in $i = x$ or y direction
ε_{st}^i	is the total strain at the stirrup in $i = x$ or y direction
ε_{su}^i	is the ultimate strain in the transverse steel reinforcement in $i = x$ or y direction
θ_p	is the angle of the compression strut with respect to the column longitudinal axis
ρ_s	is the volumetric ratio of the transverse reinforcement
ϕ_y	is the yielding curvature
ϕ_p	is the plastic portion of the curvature
$\sigma_{e,x}$	is the confinement stress in the X direction
$\sigma_{e,y}$	is the confinement stress in the Y direction
σ_{sv}	is the stress in the transverse reinforcement from the shear force
$\sum \frac{A_{st}}{s}$	is the area of the transversal reinforcement per unit length
ψ_u	is the ultimate dilation ratio

2. Effects of shear forces at material level

In a reinforced concrete column, both the longitudinal and transverse reinforcements provide resistance to the flexural, axial and shear forces simultaneously. The transverse reinforcement resists the shear force and confines the concrete core, thereby increasing its strength and ductility. Similarly, the longitudinal reinforcement and the concrete participate in the shear resisting mechanism. All of these mechanisms interact with each other such that the degradation of one mechanism affects the other mechanisms [10].

2.1. Effects of shear forces on confinement stresses

Currently, in the seismic design or assessment of a reinforced concrete column, the compressive strength of the confined concrete in a column is determined by assuming that the column is subjected only to a compression force (see Fig. 1a) using Mander's model [11]. In this study, we suggest that the confinement stress is reduced by the presence of a concurrent shear force, as shown in Fig. 1b. Consequently, the effective confinement stresses in two directions $(\sigma_{e,x}, \sigma_{e,y})$ can be calculated using Eqs. (1) and (2):

$$\sigma_{e,x} = \alpha \frac{2 \cdot A_{st} \cdot (f_{yt} - \sigma_{sv})}{s \cdot D_c} \tag{1}$$

$$\sigma_{e,y} = \alpha \frac{2 \cdot A_{st} \cdot f_{yt}}{s \cdot D_c} \tag{2}$$

where A_{st} is the area of the stirrups or the spiral; f_{yt} is the yielding strength of the stirrups; σ_{sv} is the stress in the transverse reinforcement from the shear force; s is the stirrups spacing; D_c is the center-to-center diameter of the stirrups; and α is a factor that represents the confinement efficiency, which can be obtained using Mander's model [11].

Here, the confinement stresses $(\sigma_{e,x}, \sigma_{e,y})$ are used to obtain the characteristics of the stress-strain curve of the confined concrete. Recall that the asymmetric confinement stresses provide the confined concrete with less confinement capacity than that provided by symmetric confinement stresses, as shown in Fig. 2.

2.2. Effect of bidirectional shear forces on concrete ductility

In the seismic prediction response of a reinforced concrete column, the ultimate strain in confined concrete has been obtained taking into account that the column is under axial load with empirical [12–15] or energy balance approaches [11,16]. In this study, experimental observations showed that under cyclic shear forces applied in two directions (X and Y), the strains induced by the shear forces (ε_{sv}^j) at the transverse reinforcement accumulate with the strains produced by the dilatancy of concrete in compression (ε_{sc}^i), as shown in Fig. 3. Consequently, the total strain at the stirrup (ε_{st}^i) is given by Eq. (3). The superscripts $(iorj)$ denote the direction x or y evaluated, respectively.

$$\varepsilon_{st}^i = \varepsilon_{sv}^j + \varepsilon_{sc}^i \tag{3}$$

There are two ways of quantitatively interpreting the equation given above.

The effective deformation capacity for confinement (ε_{sc}^i) is considered to be the ultimate strain in the stirrup (ε_{su}^i) minus the strain produced by the shear forces (ε_{sv}^j), as shown in Eq. (4):