Engineering Structures 134 (2017) 334–345

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Performance-based optimization of nonlinear structures subject to stochastic dynamic loading

Jiaqi Xu [a], Billie F. Spencer Jr. [b,*], Xilin Lu [a]

[a] State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China

[b] Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, 205 North Mathews Avenue, MC-250, Urbana, IL 61801, USA

ARTICLE INFO

Article history:
Received 23 March 2016
Revised 15 August 2016
Accepted 26 December 2016

Keywords:
Performance-based optimization
Structural optimization
Nonlinear structure
Stochastic dynamic loading

ABSTRACT

Structural optimization has been shown to be an efficient and effective method to obtain the optimal design balancing competing objectives. However, literature on optimization of structures subject to random excitation is sparse. This study proposes a performance-based optimization approach for nonlinear structures subject to stochastic dynamic excitation. The optimization procedure is formulated as a multi-objective problem considering various performance objectives. The excitation is modeled as a zero-mean filtered white noise and combined with the nonlinear equations of motion of the structure to create an augmented state space representation of the system. The optimization objectives are defined in terms of the variance of stationary structural responses, which are obtained via equivalent linearization. Thus, the stochastic optimization problem is converted into its deterministic counterpart. Numerical examples are provided to demonstrate the efficacy of the proposed approach. Three levels of seismic magnitudes, i.e., low-level, frequent earthquake, medium-intensity earthquake and high-intensity earthquake, are investigated. For each seismic magnitude, two performance objectives are considered. The first performance objective considers serviceability, seeking to minimize floor acceleration response; and the second performance objective considers structural safety and seeks to minimize interstory drift response. The Pareto optimal fronts are calculated to illustrate the intrinsic tradeoffs between serviceability and safety of designs subject to all seismic magnitudes.

1. Introduction

Traditional trial-and-error structural design methods rely mainly on experience and experimental observation, which cannot guarantee an optimal solution [1]. Structural optimization has been shown to be an efficient and effective method to balance competing design objectives. While many of the loads considered in structural design are stochastic and dynamic in nature, e.g., wind and seismic excitation, most of the research on structural optimization focuses on static loads or represents dynamic excitations by equivalent static loads [2,3].

Previous research on structural optimization subject to stochastic loading has been conducted using Monte Carlo simulation (MCS) [4,5]. Time history analysis can describe structural behavior in time domain. However, a major drawback of the MCS is that structural optimization usually requires numerous simulations to get converged response statistics. Structural optimization employing MCS can be prohibitively time consuming and computationally demanding.

Several researchers have conducted structural optimization applying random vibration theory. Jensen and Sepulveda [6] used a modal-based approach to optimize element sizes for a linear five-story shear structure subject to stochastic seismic ground motion. Jensen [7] continued this study carrying out structural optimization with deterministic linear dynamic systems. Then Jensen [8] extended the study to nonlinear structures using equivalent linearization. More recent attention by other researchers [9,10] focused on structural optimization subject to stochastic seismic and wind excitation. The main limitation of these studies, however, is that only drift minimization (i.e., structural safety) is considered, while other competing performance indicators (e.g., serviceability) are ignored.

A large body of literature has been published on performance-based design (PBD) since Krawinkler [11] first proposed this concept. Compared with traditional structural design methods, PBD obtains explicit evaluation of structural responses subject to predefined performance objectives. Structural performances are often divided into two main categories: safety and serviceability. By

* Corresponding author.
E-mail address: bfs@illinois.edu (B.F. Spencer Jr.).

http://dx.doi.org/10.1016/j.engstruct.2016.12.051

combining different safety and serviceability requirements, design tradeoffs can be assessed. Within plethora of response parameters, interstory drift and acceleration are most frequently considered [12]. Interstory drift response is related to damage level, i.e., safety, and acceleration is often related to serviceability [13].

Structural optimization was applied to PBD [14–16] soon after the PBD method was proposed, and the concept was referred as performance-based optimization (PBO). PBO focuses on obtaining optimum structural designs considering different performance objectives, which can provide a valuable reference for the designers. The PBO design is mainly conducted for earthquake engineering [12,17], but recent research efforts have applied this concept for other design loadings [9,14,18]. Nevertheless, structural optimization subject to stochastic dynamic loading in the PBO perspective appears to be lacking in the literature.

This study proposes a PBO procedure for nonlinear structures subject to stochastic dynamic excitation, which is formulated as a multi-objective optimization problem. The excitation is modeled as a zero-mean filtered white noise and creates an augmented state space representation of the system combining with the nonlinear equations of motion of the structure. The optimization objectives are defined in terms of the variance of stationary structural responses. Illustrative examples are presented for three levels of seismic excitation, representing frequent, medium-intensity and high-intensity earthquakes. For each seismic magnitude, structural safety and serviceability are examined, and the Pareto optimal front is employed to illustrate the tradeoff between the two competing objectives. These results demonstrate the efficacy of the proposed PBO approach.

2. Problem formulation

This section presents the formulation of the proposed PBO procedure. The nonlinear structure is cast into the first-order ordinary differential equations and subsequently linearized. The structure is excited by a stationary filtered white noise representing stochastic excitation. Structural parameters are referred as the design variables. The optimization objectives are defined in terms of performance indicators, which are represented by stochastic structural responses (function of design variables).

2.1. Structural model description

Considering hysteretic behavior, the equation of motion (EOM) of a N-degree of freedom (NDOF) nonlinear system is given by

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{C}\dot{\mathbf{d}} + \mathbf{r}(t) = \mathbf{G}\mathbf{p}(t) \tag{1}$$

where $\mathbf{M}$ is the mass matrix; $\mathbf{C}$ is the linear damping matrix; $\mathbf{d}$ is the interstory drift vector between consecutive floors; $\mathbf{r}(t)$ represents the restoring force vector; $\mathbf{p}(t)$ is the input excitation vector; and $\mathbf{G}$ is a matrix coupling the excitation dimension and the structural DOFs. Note that although the vector $\mathbf{r}(t)$ and $\mathbf{p}(t)$ are represented by the time domain symbol in Eq. (1), MCS is not required; rather, this study proposes an analytical procedure to determine the second-order response statistics that are used in the optimization procedure.

In this study, the Bouc-Wen model [19,20] is adopted to describe the hysteretic restoring force, $\mathbf{r}(t)$, i.e.,

$$\mathbf{r}(t) = \alpha\mathbf{K}\mathbf{d} + (1-\alpha)\mathbf{K}d_y\mathbf{z}(t) \tag{2}$$

where α is the rigidity ratio, i.e., the post- to pre-yield stiffness ratio; $\mathbf{K}$ is the linear stiffness matrix; d_y is the yield drift; and $\mathbf{z}(t)$ is an evolutionary vector representing the hysteretic component of the restoring force. The evolutionary variable associated with the ith story, z_i, can be described by the differential equation [21]

$$\dot{z}_i = \frac{1}{d_y}\left(-\gamma|\dot{d}_i||z_i|^{n-1}z_i - \beta\dot{d}_i|z_i|^n + A\dot{d}_i\right) \tag{3}$$

where A, γ and β are the shape coefficients of the hysteresis loop. n governs the smoothness of the transition part from elastic to plastic response. d_i is the interstory drift of the ith story. For more information about the use of the Bouc-Wen model to represent nonlinear structural response, the reader is directed to the following reference [22–24].

Combining Eqs. (1) and (2), the EOM becomes

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{C}\dot{\mathbf{d}} + \alpha\mathbf{K}\mathbf{d} + (1-\alpha)\mathbf{K}d_y\mathbf{z}(t) = \mathbf{G}\mathbf{p}(t) \tag{4}$$

To fit the state space formulation and determine the stochastic responses, the structural system must be linearized. Eq. (4) is already linear equation, so only the equation governing the evolutionary vector $\mathbf{z}$ (Eq. (3)) needs to be linearized, which is given by

$$\dot{\mathbf{z}} + \mathbf{C}_{\text{eq}}\dot{\mathbf{d}} + \mathbf{K}_{\text{eq}}\mathbf{z} = \mathbf{0} \tag{5}$$

where $\mathbf{C}_{\text{eq}}$ and $\mathbf{K}_{\text{eq}}$ are the linearized coefficient matrices. Assuming a zero-mean stationary Gaussian excitation and an EOM satisfying smoothness requirements, the mean square error is minimized when conducting equivalent linearization of the evolutionary vector to yield [21,25,26]

$$\mathbf{K}_{\text{eq},ij} = \mathrm{E}\left[\frac{\partial r_i}{\partial d_j}\right], \quad \mathbf{C}_{\text{eq},ij} = \mathrm{E}\left[\frac{\partial r_i}{\partial \dot{d}_j}\right] \tag{6}$$

where $\mathrm{E}[\cdot]$ is the expectation operator. The equivalent linear matrices $\mathbf{C}_{\text{eq}}$ and $\mathbf{K}_{\text{eq}}$ for the Bouc-Wen model were evaluated in [21,25]. For $n = 1$, when $i = j$,

$$\begin{aligned} \mathbf{C}_{\text{eq},ii} &= \tfrac{1}{d_y}\left(\gamma\mathrm{E}\left[z_i\tfrac{\partial|\dot{d}_i|}{\partial\dot{d}_i}\right] + \beta\mathrm{E}[|z_i|] - A\right) \\ \mathbf{K}_{\text{eq},ii} &= \tfrac{1}{d_y}\left(\gamma\mathrm{E}\left[|\dot{d}_i|\right] + \beta\mathrm{E}[\dot{d}_i\tfrac{\partial|z_i|}{\partial z_i}]\right) \end{aligned} \tag{7}$$

When $i \neq j$, $\mathbf{C}_{\text{eq},ij} = 0$ and $\mathbf{K}_{\text{eq},ij} = 0$. The equivalent linear responses, $\dot{d}_i$ and z_i, are jointly Gaussian subject to Gaussian excitation, and the matrices $\mathbf{C}_{\text{eq}}$ and $\mathbf{K}_{\text{eq}}$ can be evaluated in terms of the second moments of $\dot{\mathbf{d}}$ and $\mathbf{z}$ [21], i.e.,

$$\begin{aligned} \mathbf{C}_{\text{eq},ii} &= \tfrac{1}{d_y}\left(\sqrt{\tfrac{2}{\pi}}\left[\gamma\tfrac{\mathrm{E}[\dot{d}_iz_i]}{\sigma_{\dot{d}_i}} + \beta\sigma_{z_i}\right] - A\right) \\ \mathbf{K}_{\text{eq},ii} &= \tfrac{1}{d_y}\left(\sqrt{\tfrac{2}{\pi}}\left[\gamma\sigma_{\dot{d}_i} + \beta\tfrac{\mathrm{E}[\dot{d}_iz_i]}{\sigma_{z_i}}\right]\right) \end{aligned} \tag{8}$$

where $\sigma(\cdot)$ represents the standard deviation. Note that Eq. (8) is the expression for the special case of $n = 1$. Expression for more general cases ($n \neq 1$) are also reported by [21].

2.2. State space formulation

Defining a state vector $\mathbf{x}_s$ as

$$\mathbf{x}_s = \left(\mathbf{d}^{\mathrm{T}} \quad \dot{\mathbf{d}}^{\mathrm{T}} \quad \mathbf{z}^{\mathrm{T}}\right)^{\mathrm{T}} \tag{9}$$

then the equivalently linearized structure can be modeled as

$$\begin{aligned} \dot{\mathbf{x}}_s &= \mathbf{A}_s\mathbf{x}_s + \mathbf{B}_s\mathbf{p}(t) \\ \mathbf{y}_s &= \mathbf{C}_s\mathbf{x}_s + \mathbf{D}_s\mathbf{p}(t) \end{aligned} \tag{10}$$

where $\mathbf{y}_s$ is the response vector; $\mathbf{A}_s$ and $\mathbf{B}_s$ are the state space matrices, which can be written as

$$\mathbf{A}_s = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -\alpha\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} & -(1-\alpha)d_y\mathbf{M}^{-1}\mathbf{K} \\ 0 & -\mathbf{C}_{\text{eq}} & -\mathbf{K}_{\text{eq}} \end{bmatrix}, \quad \mathbf{B}_s = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{G} \\ 0 \end{bmatrix} \tag{11}$$