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# Vibration-based damage detection for structural connections using incomplete modal data by Bayesian approach and model reduction technique

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ABSTRACT

Most of the existing damage detection methods focused on damage along members of the structure without considering possible damage at its connections. Under the Bayesian framework, the finite element (FE) model reduction technique and the system mode concept, this paper presents a practical method for structural bolted-connection damage detection using noisy incomplete modal parameters identified from limited number of sensors. Based on the incomplete modal identification results, the most probable structural model parameters, the most probable system eigenvalues and partial modes shapes together with the associated uncertainties can be identified simultaneously. There are several significant features of the proposed method: (1) it does not require computation of the system mode shapes for the full model due to the FE model reduction technique; (2) matching between measured modes and model predicted modes is avoided in contrast to most existing methods in the literature; and (3) an efficient iterative solution strategy is also proposed to resolve the difficulties arisen from the high-dimensional nonlinear optimization problem for the structural model parameters and the incomplete system modal parameters. Numerical simulations and experimental verifications of a four-storey two-bay bolt-connected steel frame and a two-storey laboratory bolted frame, respectively, are utilized to demonstrate the validity and efficiency of proposed methodology.



## 1. Introduction

Over the last few decades, there has been great effort in developing structural health monitoring (SHM) methodologies utilizing vibration measurements [1,2]. Most of the methods in the literature have been verified by various types of structural components or systems, such as truss-type structures [3–5], beam-type structures [6–9], railway sleepers [10,11], plates [12–15], frame structures [16–19], and shear building models [20–24]. The majority of these mentioned damage detection methods are based on modal parameters, where an objective function is usually defined in terms of the discrepancies between the experimental modal parameters and those calculated from a FE model by assuming that the influence of damage on structural mass can be neglected, and it is then minimized for the estimation of the stiffness parameter changes. However, this type of damage identification procedures generally requires solving the eigensystem equation repeatedly to compute the model output and the objective function at least once for each iteration step, which is extremely time-consuming especially for large-scaled structural models. In addition, in the formation of the above objective function, it is necessary to ensure that the measured modes are matched well with the calculated ones by using the modal assurance criterion (MAC) technique but it is difficult in real application due to limited number of measured degrees of freedom (DOFs). Moreover, since damage might cause changes in the order of the modes, such mode matching becomes even more difficult.

For this reason, some methods have been proposed to avoid the above-mentioned mode matching problem by introducing the concept of system mode shapes, which is distinct from those calculated from the structural model specified by some given model parameters. Some of these methods employ Rayleigh quotient frequencies, which are derived from the structural model and the system mode shapes, to avoid repeatedly solving the eigensystem equation during damage detection process. Then, the structural model parameters and the system mode shapes are identified

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simultaneously [20,25,26]. Later, an improved method has been proposed so that the system eigenvalues are also included as additional unknown parameters to be identified based on the incomplete modal data besides of the system mode shapes in order to represent the actual modal frequencies of the structural system [27,28].

It is noted that, in the above-mentioned methods utilizing the concept of system modes, the system mode shapes with respect to the full structural models are required. In real situations, mode shapes are usually measured with incomplete components or missing DOFs. In order to resolve the difficulties arisen from the limited number of measurement channels, the methods usually start from computing the missing components of the mode shapes through mode shape expansion procedure [27]. However, it has been revealed that this would aggregate the modeling error, experimental noise and other sources of uncertainties into the resultant mode shapes [29,30], affecting substantially the damage detection results. On the other hand, the dimension of full system mode shapes and, thus, the number of the unknown parameters becomes extremely large, resulting in unreliable or even unidentifiable damage detection results especially for large-scaled structures. In such circumstances, the FE model reduction method originally developed for the purpose of reducing the computation effort for large-scaled structural models [31–34], particularly for the dynamic-reduction method [34], becomes a more practical alternative since it does not introduce any error in the transformation process within a certain frequency range [35,36]. Using the dynamic model reduction technique, the unknown full system mode shapes correspond only to the condensed structural model so the dimension of the inverse problem could be significantly reduced, especially efficient for large-scaled complex structural models with a huge number of DOFs.

On the other hand, majority of existing damage detection methods concentrated on damage along members of the structure without consideration of connection damages, which frequently occur in structural frames. In this paper, by acknowledging this importance and the aforementioned difficulties, the system mode based damage detection method [27,28] is improved by involving the efficient dynamic model reduction based method [35,36] to become a more practical and workable method to detect structural connection damage for frame-type structures. The proposed method can handle the real situations with very limited number of sensors available and has the particular potential for large-scaled complex structures due to the model reduction strategy. In the proposed method, the uncertainty issues associated with the unknown structural model parameters and system modal parameters are well treated by the Bayesian probabilistic approach. In addition, the FE model reduction technique is also utilized to construct the prior PDF of the incomplete system modal parameters by evaluating the compatibility of the system modal parameters to the reduced structural model.

One of the most attractive features of the proposed methodology is that the identification of full system mode shapes and, hence, mode matching are completely avoided. Furthermore, an efficient iterative solution method is also presented to solve the optimization problem for the most probable values of the structural model parameters and the incomplete system modal parameters. After presenting the theoretical development in detail, the proposed methodology is validated and demonstrated thoroughly by a comprehensive set of numerical case studies of a four-storey two-bay bolt-connected steel frame and experimental verification of a laboratory two-storey bolted frame with both single- and double-damage situations.

## 2. Theoretical developments

### 2.1. Dynamic reduction of the eigen-system equations

Consider a class of structural models $\mathcal{M}$ discretized by FE method into $N$ DOFs. Assuming that the change of mass matrix due to possible damage is negligible, the corresponding eigen-system equation is thus given by:

$$\mathbf{K}(\boldsymbol{\theta})\boldsymbol{\phi}_j = \lambda_j \mathbf{M}\boldsymbol{\phi}_j \tag{1}$$

where $\lambda_j, \boldsymbol{\phi}_j \in R^N$, $j = 1, 2, \ldots, N_t$, are the $j$th eigenvalue and eigenvector, respectively. $N_t$ is the number of measured modes. The global stiffness matrix $\mathbf{K}(\boldsymbol{\theta})$ is parameterized by the stiffness scaling parameter vector $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_{N_\theta}]^{\mathrm{T}} \in R^{N_\theta}$:

$$\mathbf{K}(\boldsymbol{\theta}) = \mathbf{K} - \sum_{i=1}^{N_\theta} \theta_i \mathbf{K}_{(i)} \tag{2}$$

where the stiffness scaling parameters $\boldsymbol{\theta}$ allow the nominal stiffness matrix given by $\boldsymbol{\theta} = \boldsymbol{\theta}_0$ in Eq. (2) to be updated based on the identified modal parameters from the structure. $\mathbf{K}_{(i)}$, $i = 1, 2, \ldots, N_\theta$ is the contribution of the $i$th member or substructure to the global stiffness matrix of the FE model; and $N_\theta$ is the number of unknown stiffness scaling parameters to be identified.

By using Eq. (2), Eq. (1) can be rearranged to partition the measured and unmeasured DOFs as follows:

$$\begin{bmatrix} \mathbf{K}_{mm} - \sum_{i=1}^{N_\theta}\theta_i[\mathbf{K}_{(i)}]_{mm} & \mathbf{K}_{ms} - \sum_{i=1}^{N_\theta}\theta_i[\mathbf{K}_{(i)}]_{ms} \\ \mathbf{K}_{sm} - \sum_{i=1}^{N_\theta}\theta_i[\mathbf{K}_{(i)}]_{sm} & \mathbf{K}_{ss} - \sum_{i=1}^{N_\theta}\theta_i[\mathbf{K}_{(i)}]_{ss} \end{bmatrix} \begin{Bmatrix} \{\boldsymbol{\phi}_m\}_j \\ \{\boldsymbol{\phi}_s\}_j \end{Bmatrix} = \lambda_j \begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{sm} & \mathbf{M}_{ss} \end{bmatrix} \begin{Bmatrix} \{\boldsymbol{\phi}_m\}_j \\ \{\boldsymbol{\phi}_s\}_j \end{Bmatrix} \tag{3}$$

where $\{\boldsymbol{\phi}_m\}_j$ and $\{\boldsymbol{\phi}_s\}_j$ are the measured and unmeasured parts of full mode shape $\boldsymbol{\phi}_j$ for the $j$th mode, with dimensions $N_m$ and $N_s$, respectively, and $N_m + N_s = N$.

The lower portion of Eq. (3) can be rewritten as:

$$\{\boldsymbol{\phi}_s\}_j = \mathbf{D}_j\{\boldsymbol{\phi}_m\}_j \tag{4}$$

and

$$\mathbf{D}_j = -\mathbf{F}_j^{-1}\mathbf{G}_j \tag{5}$$

where $\mathbf{D}_j$ is the dynamic reduction matrix for the $j$th mode, and it is the function of eigenvalue $\lambda_j$ and the stiffness scaling parameters $\boldsymbol{\theta}$, and

$$\begin{aligned} \mathbf{F}_j &= \mathbf{K}_{ss} - \sum_{i=1}^{N_\theta}\theta_i[\mathbf{K}_{(i)}]_{ss} - \lambda_j\mathbf{M}_{ss} \\ \mathbf{G}_j &= \mathbf{K}_{sm} - \sum_{i=1}^{N_\theta}\theta_i[\mathbf{K}_{(i)}]_{sm} - \lambda_j\mathbf{M}_{sm} \end{aligned} \tag{6}$$

Thus, for the $j$th mode, the full mode shape can be represented only by the corresponding measured part as

$$\begin{bmatrix} \{\boldsymbol{\phi}_m\}_j \\ \{\boldsymbol{\phi}_s\}_j \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{N_m} \\ \mathbf{D}_j \end{bmatrix}\{\boldsymbol{\phi}_m\}_j = \mathbf{T}_j\{\boldsymbol{\phi}_m\}_j \tag{7}$$

where $\mathbf{T}_j \in R^{N\times N_m}$ is the transformation matrix of the $j$th mode, and $\mathbf{I}_{N_m}$ is the $N_m \times N_m$ identity matrix.

Substituting Eq. (7) into Eq. (3), and pre-multiplying transpose of $\mathbf{T}_j$ to the both sides of the resultant equation set, the eigen-system equation of the reduced FE model corresponding to the $N_m$ measured DOFs is obtained:

$$\overline{\mathbf{K}}_j^{\mathrm{R}}\{\boldsymbol{\phi}_m\}_j = \lambda_j\mathbf{M}_j^{\mathrm{R}}\{\boldsymbol{\phi}_m\}_j \tag{8}$$