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Second-order flexibility-based model for nonlinear inelastic analysis of 3D semi-rigid steel frameworks

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ABSTRACT

This paper presents an efficient computer method for large deflection distributed plasticity analysis of 3D semi-rigid steel frameworks. A novel second-order inelastic flexibility-based element has been developed by combining the Maxwell-Mohr rule and the second-order force based functions for computation of the generalized displacements. The proposed model allows explicit and efficient modelling of the initial geometric imperfections and residual stresses and is intended to model the combined effects of nonlinear geometrical and gradual spread-of-plasticity by using only one beam-column element per physical member. At the cross-sectional level the proposed method addresses computational efficiency through the use of path integral approach to numerical integration of the cross-sectional nonlinear characteristics and residual stresses, enabling in this way the accurate geometrical specifications and precise modelling of cross-sections. The combined effects of material, geometric and connection behavior nonlinearity sources have been implemented in a general nonlinear static purpose computer program. Several computational examples are given to validate the effectiveness of the proposed method.

1. Introduction

Geometric nonlinearity due to the large and small P-Delta effects, material inelasticity and semi-rigid beam-to-column connection behavior are the most important nonlinear sources exhibited by steel frameworks. In addition, the geometrical and material (i.e. residual stresses) imperfections are very difficult to alleviate during manufacturing process of real structures. These imperfections affect both the load carrying capacity and deformability of steel structures and must be considered in an advanced nonlinear inelastic analysis procedure [1,2].

There currently exist several methods and computer programs with an emphasis to the nonlinear inelastic analysis of frame structures with rigid and semi-rigid connections. At one extreme, two- and three dimensional finite elements enhanced with advanced material constitutive laws were used to investigate the nonlinear response of steel frameworks with rigid and semi-rigid connections (e.g., [3,4]). All these available computer programs and numerical models for such advanced analyses are general purpose FE programs that require very fine-grained modelling, extensive calibration and mesh generation studies that are often impractical for current design practice. At the other extreme, the line elements (1-D) approach in conjunction with either *distributed* or *concentrated* plasticity models, have been devoted to the development of nonlinear analysis tools that provide a desirable balance between accuracy and computational efficiency (e.g., [5–31]) among others. Within the context of the line finite elements (1-D), there are three main approaches that have been used to model the gradual plastification and spread of plasticity (distributed plasticity) in a nonlinear inelastic analysis [5], one based on the displacement method (e.g., [6–8]), the second one based on the force or flexibility method (e.g., [9–11]), and the third one refers to mixed or hybrid approach (e.g., [12,13]). In spite of the simplicity and ease of implementation, because classical displacement-based finite elements (i.e. the displacement field is approximated with lower-order Hermitian shape functions) implicitly assumed linear curvatures along the element length, accuracy in this approach, when geometrical and/or material nonlinearity is taken into account, can be obtained only using several elements in a single structural member. Thus the computational effort is greatly enhanced and the method becomes prohibited computational in the case of large scale frame structures. On the other hand in the either flexibility or mixed based approach only one element per physical member can be used to simulate the gradual spread of yielding throughout the volume of the members, but the complexity of these methods derives from their implementation in a finite element analysis program (i.e. state determination procedure) and the inclusion of the element geometrical effects [27,28].

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In the concentrated plasticity approach [20–26] which is usually based on the plastic hinge concept, with different degrees of refinement, the effect of material yielding is "lumped" into a dimensionless plastic hinge. In the plastic hinge locations if the cross-section forces are less than cross-section plastic capacity, either elastic behavior or gradual transition (refined plastic hinge) from elastic to plastic behavior is assumed. The plastic hinge approach could eliminate the integration process on the cross section and permits the use of fewer elements for each member, and hence greatly reduces the computing effort. Unfortunately, as plastification in the member is assumed to be concentrated at the member ends, the plastic hinge model is usually less accurate in formulating the member stiffness, requires calibration procedures, but make possible to use least number of elements per physical member to simulate geometric and material nonlinearities in building frameworks [20–24].

In the efforts to develop an intermediate solution that has the computational efficiency of plastic hinge methods and the accuracy of distributed plasticity methods several researchers developed quasi-plastic hinge [14–19,29–31], stress-resultant constitutive models [25,26] and higher order refined plastic hinge elements [23,24]. Although subject to some limitations of required calibration these methods have been shown to make distributed plasticity analyses practical for large scale 3D steel [14–16,18–22] and composite steel–concrete frameworks [17,23,24], using least number of elements per structural member to capture the nonlinear behavior of frame structures.

In spite of the availability of such nonlinear inelastic algorithms and powerful computer programs, the nonlinear inelastic analysis of real large-scale 3D semi-rigid frame structures still possess high demands on the most powerful computers available and still represents unpractical tasks to most designers.

The present work attempts to develop accurate yet computational efficient tools for the nonlinear inelastic analysis of 3D steel frameworks with semi-rigid connections fulfilling the practical and advanced analysis requirements. Within the framework of flexibility-based formulation a 3D frame element with 12 DOF able to take into account gradual softening and spread of plasticity, second-order geometrical effects, initial geometrical and material imperfections and connection semi-rigidity is developed. The present paper extends the previous study of the author in [15–17] in several fundamental respects: (i) A novel second-order inelastic flexibility-based element has been developed by coupling consistently at the element level the inelastic behavior, second-order effects due to element deformation (P-delta effect) and initial geometric imperfections. The incremental force-displacement relationships are derived by applying the Maxwell-Mohr rule in conjunction with the second-order force based functions for computation of generalized displacements in the second-order geometrically nonlinear analysis. In the previous formulations [15–17] the incremental force-displacement relationships at the element level have been derived considering the *geometric linear flexibility formulation* and correcting the resulting stiffness matrix and equivalent nodal forces with stability stiffness functions in order to include the P-delta effects (i.e. combine the use of both flexibility and stiffness approach). As a consequence there is a lack of consistency between the inelastic behavior formulation (i.e. force-based approach) and how the element geometrical effects are included (i.e. displacement-based approach). In the present formulation this lack of consistency is eliminated leading to a full force-based formulation but keeping the computational efficiency of the previous approaches [15–17]; (ii) A new force recovery procedure to find the nodal displacements, element resisting forces and efficient modelling of initial geometric imperfections is implemented corresponding to the proposed second-order flexibility based model; and (iii) At the cross-sectional level the proposed method addresses computational efficiency and modelling shortcomings through the use of path integral approach to numerical integration of the cross-sectional nonlinear characteristics and residual stresses without the need to decompose the cross-section in distinct regions according with changes in the definition of the residual stress distribution. In this way the difficulties of integration on separate regions over entire cross-section is avoided, by integrating only on the positively oriented boundary of cross-section, leading to less computational effort since it does not require the subdivision of cross-section into several regions as in [17] and allows efficiently to handle various circular shapes such as fillet regions which define the exact geometry of the structural steel profiles.

Comparing the proposed method with the related second-order distributed plasticity methods developed in [10,11,28] the present approach has several features that make the proposed element more practical in the context of implementation in finite element analysis program and possess accuracy comparable to that of fibre-flexibility or fibre-displacement finite elements. As will be briefly described in the following sections, the incremental force-displacement relationships at the element level are derived directly from energetic principles, by applying the Maxwell-Mohr rule for computation of generalized displacements in the second-order geometrically nonlinear analysis. In this respect the element force fields are described by the second-order bending moments and shear forces derived by solving the second-order differential equilibrium equation expressing the variation of the bending moment along the member length in the presence of the compressive axial force (P-delta effects), member lateral loads and the second-order effects associated with the initial geometric imperfections. Thus, at least in the elastic domain, the element force fields can be exactly described as function of the nodal and applied element forces and hence the element nonlinear geometrical effects are included *directly* in the proposed formulation by using nonlinear functions of axial force. On the other hand in the displacement-based formulations, the deformed shape of the element is obtained directly based on the nodal displacement values and the adopted shape functions. Thus, the implementation of the element second-order effects is straightforward, but the accuracy is dependent by the number of the finite elements involved. In the flexibility based formulations [10,11,28] there are no deformation shape functions to relate the deformation field inside the element to the nodal displacements, hence, more elaborate and more time consuming *indirect* procedures are required. For instance, in order to capture the element geometrical effects in [28] is developed a curvature-based displacement interpolation (CBDI) function whereas in [10] Simpson integration scheme along with piecewise interpolation of the curvature is applied. In the proposed approach because the force fields are described by exact solutions of the second-order differential equilibrium equation, the modelling and solution time is minimized and generally only one element are needed per member in order to simulate the second order effects and no approximations of the curvatures along the element length are involved to approximate the second-order elastic response. In this way the elements of the flexibility matrix and by its inversion the stiffness matrix and equivalent nodal loads can be obtained analytically and readily evaluated by computing the *correction coefficients* that affect the elastic flexibility coefficients and equivalent nodal loads. In this way numerical integrations are required only to evaluate these correction coefficients and not the entirely flexibility or stiffness matrix elements as in [10,11,28]. Besides, the effect of the transverse shear deformation can be readily included in the element formulation, both in stiffness matrix and equivalent nodal loads. However due to the fact that flexural rigidity becomes variable along the member length, as spread of plasticity develops, the second-order equilibrium