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Lower bound equilibrium element and submodel for shear joints in precast concrete structures

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ABSTRACT

This paper is concerned with the shear capacity of keyed joints reinforced with overlapping U-bar loops in the transverse direction. The layout of the loop reinforcement affects the capacity and failure mode, and currently it is not accounted for by standards or previous theoretical work. A multiscale approach to the issue is proposed: An equilibrium element for finite element limit analysis representing keyed joints is coupled with a suitable submodel, which handles the complex stress states within the joint. The submodel is based on several modified stringer models, which makes it possible to account for local mechanisms in the core of the joint. The element and submodel are validated by comparison to a detailed model based on finite element limit analysis and experimental data. The joint element and submodel lead to a small optimisation problem compared to the detailed model and the computational time is reduced by several magnitudes.

1. Introduction

The lateral stability of modern precast concrete buildings is often ensured by shear walls, i.e. precast wall panels connected by in-situ cast joints. Horizontal forces, e.g. from wind load or seismic action, are transferred as in-plane forces and the shear capacity of the panels and joints are of the utmost importance. In practice, the shear capacity of such walls is usually assessed by analytical lower bound models, e.g. strut-and-tie models or stress field methods [1,2]. The stress fields are also frequently determined by use of linear elastic finite element analysis. Naturally, this practice often leads to suboptimal structures compared to what can be obtained if the stress fields instead are determined from a non-linear elastic-plastic analysis. Use of numerical elastic-plastic analysis to determine stress fields has e.g. been demonstrated in Refs. [3,4].

The joints between the precast panels are of particular interest as they are often a critical part of the structure. In-situ cast joints consist of a concrete core and two interfaces, where the core typically is reinforced in two directions, and the interfaces typically are keyed. The shear capacity of the joints and interfaces is in practice assessed by simple empirical formulas [5] which often gives a conservative estimate of the capacity. Several authors have investigated the behaviour of in-situ cast joints. The investigations cover both experimental testing [see e.g. 6–9] and simplified mechanical models based on the theory of rigid-plasticity, namely upper bound solutions based on yield line theory [10,11] and lower bound solutions based on strut-and-tie models [1,11,12]. The experiments showed that the geometry of the joint and the reinforcement layout affect the shear capacity as well as the collapse mode, but the analytical methods have only been able to capture the observed behaviour to a certain extent. Local failure mechanisms caused by the reinforcement layout, however, have not been investigated using analytical methods. Investigations using numerical tools, e.g. finite element method or similar, have focused on single key joints often used in precast concrete segmental bridges [13,14]. These investigations have primarily been carried out by use of non-linear finite element analysis. This approach is computationally heavy, especially when considering the fact that the ultimate load carrying capacity is the result of main interest.

Herfelt et al. [15] presented a detailed model for keyed joints based on finite element limit analysis. The model was based on a lower bound formulation and the analysis yielded a statically admissible stress field. Moreover, the solution to the dual problem, i.e. the corresponding kinematic problem, was interpreted as the failure mode. The detailed model used triangular plane stress elements [16] representing the concrete, bar elements [16] representing the reinforcement, and an interface elements representing the

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concrete-to-concrete interfaces. It was shown that the model could represent the complex stress states within the joint and captured the local failure mechanisms to a satisfactory degree; however, for practical design it is not feasible to use that level of detail. Fig. 1 shows a four storey wall comprising several precast panels connected by in-situ cast joints. As indicated in the figure, plane stress elements may be used to model the precast panels, while a special joint element is needed for the joints.

This paper presents a lower bound equilibrium element representing the in-situ cast joints. The element is designed for interaction with the triangular plane stress element [16] and interface elements [15]. The scope is to be able to model entire wall systems, e.g. the four storey wall seen in Fig. 1. The joint element requires a suitable yield criterion which can capture the critical mechanisms identified by the detailed model [15], and for this purpose, a semi-analytical submodel yield criterion based on the stringer method is developed. The joint element and submodel fit the format of second-order cone programming, and the developed model is compared to the detailed model [15] as well as experimental data [6,7]. The proposed multiscale model captures the behaviour of the detailed model as well as the specimens.

2. Problem formulation

Finite element limit analysis can be considered as a special case of the general finite element method: It is based on the extremum principles for rigid-plastic materials [see e.g. 1,17,18] and deploys a mesh discretisation known from the finite element method. Anderheggen and Knöpfel [19] presented a general formulation as well as equilibrium elements for solids and plates. Since the 1970s several authors have contributed to the method see e.g. [16,20,21]. Finite element limit analysis is a direct method, where the ultimate load is determined in a single step, which is a significant advantage over non-linear finite element methods for practical applications. Moreover, when modelling concrete structures, there is no need to consider any tensile strength to avoid problems related to numerical stability. From the lower bound formulation, the stress field is determined. Associated with the lower bound problem is a so-called dual problem, and the solution to this dual problem can be interpreted as the displacement field and plastic strain. Since we are dealing with a rigid plastic material model, no information on the magnitude of the strains and displacements are determined; only the directions. When the method is applied to structural concrete, it is necessary to operate with effective strength parameters (via the so-called effectiveness factors) to account for the limited ductility of concrete as well as the reduction of the compressive strength as a result of cracking and tension strains transverse to compressive stress fields. In practice, the effective strength parameters have to be found by calibration of calculations with results of tests on structural components.

Numerical lower bound limit analysis is formulated as an optimisation problem where the scope is to maximise a load factor λ. The analysis determines a statically admissible stress, i.e. a stress field which satisfies equilibrium and does not violate the yield criteria in any points. The general problem can be stated as [16,22]:

$$\begin{aligned} &\text{maximise} && \lambda \\ &\text{subject to} && \mathbf{H}\boldsymbol{\beta} = \boldsymbol{R}\lambda + \boldsymbol{R}_0 \\ &&& f(\boldsymbol{\beta}_i) \leqslant 0, \quad i = 1,2,\ldots,m \end{aligned} \tag{1}$$

The load acting on the model consists of a constant part $\boldsymbol{R}_0$ and a scalable part $\boldsymbol{R}\lambda$. The linear equality constraints ensure equilibrium while the functions $f(\boldsymbol{\beta}_i) \leqslant 0$ ensure that the stress filed does not violate the yield criteria. $\mathbf{H}$ is the global equilibrium matrix, and $\boldsymbol{\beta}$ is the stress vector. m is the number of check points for the yield function, f, which is generally convex, but non-linear; thus, (1) represents a convex optimisation problem.

In this work, the optimisation problem (1) will be a second-order cone program (SOCP). Second-order cone programming as well as semidefinite programming have been used in the field of finite element limit analysis for more than a decade, see e.g. Refs. [23–25]. Expanding the yield functions f, (1) can be restated as:

$$\begin{aligned} &\text{maximise} && \lambda \\ &\text{subject to} && \mathbf{H}\boldsymbol{\beta} = \boldsymbol{R}\lambda + \boldsymbol{R}_0 \\ &&& \mathbf{C}_\beta\boldsymbol{\beta} + \mathbf{C}_\alpha\boldsymbol{\alpha} + \mathbf{C}_\gamma\boldsymbol{\gamma} = \boldsymbol{C}_0 \\ &&& \mathbf{E}_\beta\boldsymbol{\beta} + \mathbf{E}_\alpha\boldsymbol{\alpha} + \mathbf{E}_\gamma\boldsymbol{\gamma} \leqslant \boldsymbol{E}_0 \\ &&& \boldsymbol{\gamma}_i \in \mathcal{Q}_{k_i}, \quad i = 1,2,\ldots,m \end{aligned} \tag{2}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ are problem variables associated with the yield functions, and the $\mathbf{C}$ and $\mathbf{E}$ matrices define the necessary linear equalities and inequalities for the chosen yield criterion. The variables $\boldsymbol{\gamma}_i$, associated with the ith check point for the stresses, are in a quadratic cone $\mathcal{Q}_{k_i}$ of size k_i, defined as:

$$\mathcal{Q}_k = \left\{ \boldsymbol{x} | \boldsymbol{x} \in \mathbb{R}^k,\ x_1 \geqslant \sqrt{x_2^2 + \cdots + x_k^2} \right\} \tag{3}$$

The problem (2) can be solved efficiently using interior point algorithm, and in this work the commercial solver MOSEK [26] is used. The reader is referred to Refs. [27–30] for a detailed description of SOCP and interior point solvers.

On the element level, the equilibrium equations and yield functions can be stated as:

$$\begin{aligned} &\mathbf{h}_{el}\boldsymbol{\beta}_{el} = \boldsymbol{q}_{el} \\ &\mathbf{C}_\beta^{el,i}\boldsymbol{\beta}_{el} + \mathbf{C}_\alpha^{el,i}\boldsymbol{\alpha}_i + \mathbf{C}_\gamma^{el,i}\boldsymbol{\gamma}_i = \boldsymbol{C}_0^{el,i}, \quad i = 1,2,\ldots,m_{el} \\ &\mathbf{E}_\beta^{el,i}\boldsymbol{\beta}_{el} + \mathbf{E}_\alpha^{el,i}\boldsymbol{\alpha}_i + \mathbf{E}_\gamma^{el,i}\boldsymbol{\gamma}_i \leqslant \boldsymbol{E}_0^{el,i}, \quad i = 1,2,\ldots,m_{el} \\ &\boldsymbol{\gamma}_i \in \mathcal{Q}_{k_i}, \quad i = 1,2,\ldots,m_{el} \end{aligned} \tag{4}$$

$\boldsymbol{q}_{el}$ is the contributions to the equilibrium equations on the global level, $\boldsymbol{\beta}_{el}$ is the stress variables of the given element, and $\mathbf{h}_{el}$ is the element equilibrium matrix. The matrices $\mathbf{C}^{el,i}$ and $\mathbf{E}^{el,i}$ define the yield function for the m_{el} check points of the element. Similarly to (2), the variables denoted $\boldsymbol{\gamma}_i$ are required to be in a quadratic cone Q_{k_i} of size k_i.

3. Keyed joints and detailed numerical model

A keyed joint reinforced with loop reinforcement (U-bars) and a locking bar is considered. Fig. 2 shows the basic geometry and a unit section (dashed rectangle) which will form the foundation of the submodel yield criterion. The thick vertical lines seen in Fig. 2 represent the loop reinforcement, while the horizontal solid line represents the locking bar. The length of keyed joints in practice usually ranges from a single storey height to the height of the entire building, while the width b and thickness t usually are below 200 mm.

For the detailed numerical model presented by Herfelt et al. [15], several thousand plane stress elements were necessary to capture the local mechanisms and stress fields developed in the core of the joint. The model was loaded such that the centre line of the joint would be subjected to pure shear, i.e. no bending. The concrete is modelled as a Mohr-Coulomb material, while a simple linear criterion is used for the rebars. For the interface, a Coulomb friction model is assumed. The model assumed plane stress state, thus, the confinement provided by the reinforcement loops and general triaxial stress states were disregarded.

Fig. 3(a) shows an example of the collapse mode determined by the aforementioned detailed model [15] using the solution to the dual problem, i.e. the corresponding kinematic problem. The inter-