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Improved displacement based alternative to force based finite element for nonlinear analysis of framed structures

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ABSTRACT

This paper presents an improved displacement based element (iDBE) for nonlinear finite element analysis of framed structures, which, like the force based elements (FBE), needs neither high order shape functions nor the refinement of the finite element mesh in straight members to attain an accurate solution. To achieve this objective, to the strain fields based on the displacements shape functions of the conventional displacement based element (cDBE) corrective fields are added, which are determined at the element level by the principle of virtual forces. Simple examples involving tapered elements, nonlinear axial-bending interaction and elasto-plastic behavior, are included to illustrate the application of iDBE and compare its performance to cDBE and FBE.

1. Introduction

In general terms, the finite element analysis of framed structures can be based on either the displacement based element (DBE) or the force based element (FBE). The former is the most popular approach due (i) to its simpler formulation, because it is directly derived from the approximation of the displacement fields and (ii) to the more practical strain driven constitutive relations. However, when the conventional DBE (cDBE) is applied to nonlinear analysis, it shows a strong drawback, because of the poor approximation of the generalized strain fields and, thus, of the internal force fields which are based on the latter. This is an outcome of the discretization error due to the interpolation of the displacement fields, with linear and cubic polynomial functions in the common two-node finite element [13], so that the axial strain field is constant and the curvature field is linear. To improve the approximation of the generalized strain and internal force fields, the finite element mesh must be refined, increasing the dimension of the numerical model and the computation time.

The FBE does not share this drawback, because it is based on the interpolation of the internal force fields. It therefore dispenses with the mesh refinement, the solution accuracy being increased by increasing the number of integration points (NIP) in the element — this renders the FBE more efficient than the DBE [8]. The FBE is particularly advantageous when bending moment and axial force interaction is involved, e.g., in the nonlinear dynamic analysis of framed structures [13]. The FBE is developed and applied in several references, besides the two given above, e.g. [12,9,7], all with some involvement of F. Filippou.

This paper presents an improved displacement based finite element (iDBE) that gets accurate approximations to the generalized strain fields, e.g. axial strain (ε) and bending curvatures (χ), without having to employ high-order elements or reduce the finite element length. Having this objective in view, to the usual generalized strain fields, based on displacement shape functions, corrective nonlinear strain fields $\boldsymbol{\epsilon}_{\mathrm{NL},0,s}$ (defined below) are added, in analogy with the enhanced element formulation of Simo and Rifai [11]. A fundamental step in the derivation of the iDBE, is that the additional assumed strain fields must be orthogonal, at the element level, to statically admissible stress fields,

$$\int_0^L \mathbf{b}_s^{\mathrm{T}} \boldsymbol{\epsilon}_{\mathrm{NL},0,s} \mathrm{d}x = \mathbf{0} \tag{1}$$

where L is the element length and $\mathbf{b}_s$ is the matrix of shape functions for the internal force fields, commonly ignoring the element loading. Vectors and matrices are denoted by bold symbols and their dimensions are given in the notation section. This compatibility condition is like a virtual forces statement (PVF) and is so called

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in the paper. With the exception of the above condition, the general framework of the improved element is that of the conventional DBE, based on the principle of virtual displacements (PVD), see (11). The combination of the PVD and PVF statements show that this element is based on a mixed principle, but since the static parameters associated to the auxiliary stress fields are condensed at the element level, the resulting finite element can be viewed as a generalized displacement element [10]. In fact, basically, it results from adding a corrective term to the strain fields of the common DBE, and it preserves the basic structure of that element. Moreover, because of the combination of PVF and PVD, the proposed element, like the FBE, involves only a numerical integration error, and not a discretization error, meaning that mesh refinement is not needed to decrease the error [8].

2. From the fictitious forces method to the iDBE

The iDBE can be formulated either directly or as an emanation of the fictitious forces method (FFM). Choosing the latter approach one starts by presenting FFM and its application to the analysis of framed structures, under the Euler-Bernoulli beam theory. FFM [5,6], which is based on Zienkiewicz and Argyris initial stress and initial strain methods [14,1], was specifically developed as a simple iterative method for the materially nonlinear analysis. FFM operates with an auxiliary structure which is similar to the original one but (i) is made of fictitious linear elastic materials and (ii) has a diagonal linear matrix of stiffness fields, $\mathbf{K}_{\text{aux,s}}$, which is constant along each finite element and whose diagonal elements for the torsionless spatial linear element are EA_{aux}, $EI_{y,\text{aux}}$ and $EI_{z,\text{aux}}$, whereas the element stiffness matrices,

$$\mathbf{K}_{\text{aux,e}} = \int_0^L \mathbf{B}_s^{\mathsf{T}} \mathbf{K}_{\text{aux,s}} \mathbf{B}_s \mathrm{d}x \tag{2}$$

and the global stiffness matrix, $\mathbf{K}_{\text{aux}}$, are calculated as usually. In this expression, $\mathbf{B}_s$ is the matrix of strain shape functions derived from the n_d displacement shape functions (collecting these in $\mathbf{N}_s$, then $\mathbf{B}_s = \mathbf{S}\mathbf{N}_s$, where $\mathbf{S}$ is a "matrix" of differential operators [14]). The finite element is referred to a Cartesian coordinate system xyz with axis x along its longitudinal axis; the subscript $(\cdot)_s$ denotes an element field along this axis. Since the auxiliary structure is linear elastic, the stiffness matrices are calculated only once, before the iterative loop. The iDBE procedure also needs an element flexibility matrix, $\mathbf{F}_{\text{aux,e}}$, associated with the n_X element natural modes, i.e. which exclude rigid body motions [1],

$$\mathbf{F}_{\text{aux,e}} = \int_0^L \mathbf{b}_s^{\mathsf{T}} \mathbf{K}_{\text{aux,s}}^{-1} \mathbf{b}_s \mathrm{d}x \tag{3}$$

The key step of iDBE is the decomposition of the generalized strain fields in (i) what will be called their linear part, i.e. the fields corresponding to the displacement shape functions, $\mathbf{B}_s\mathbf{d}_e$, and to the effect of the actions inside the elements of the auxiliary structure with fixed nodes (vanishing nodal displacements), $\boldsymbol{\epsilon}_{0,s}^{\text{el.act.}}$, as denoted by the subscript $(\cdot)_0$, and (ii) the remaining part, denoted $\boldsymbol{\epsilon}_{\text{NL},0,s}$, which will be called their nonlinear part,

$$\boldsymbol{\epsilon}(x) \equiv \boldsymbol{\epsilon}_s = \mathbf{B}_s\mathbf{d}_e + \boldsymbol{\epsilon}_{0,s}^{\text{el.act.}} + \boldsymbol{\epsilon}_{\text{NL},0,s} \tag{4}$$

see Fig. 1. The $\boldsymbol{\epsilon}_{\text{NL},0,s}$ term in this decomposition echoes the a priori strain field assumption of the so-called assumed strain method [11].

The dimension of vector $\boldsymbol{\epsilon}_s$ is given by the number of cross-sectional independent generalized stresses ($n_{X,\text{cs}}$): for a torsionless Euler-Bernoulli spatial element $n_{X,\text{cs}} = 3$ and $\boldsymbol{\epsilon}_s = \left[\varepsilon, \chi_y, \chi_z\right]^{\mathsf{T}}$, i.e. it contains the axial strain and the two bending curvatures. The Euler-Bernoulli hypothesis determines the normal strain distribution in each section, $\epsilon_{\text{cs}}(y,z) = \varepsilon_s + z\chi_{y,s} + y\chi_{z,s} = \mathbf{l}\boldsymbol{\epsilon}_s$, the subscript

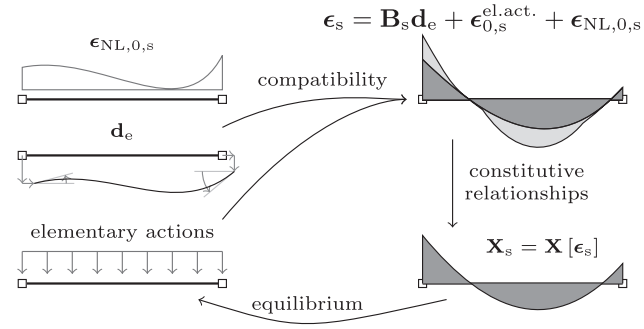

Fig. 1. Sketch of the iDBE structural relations.

$(\cdot)_{\text{cs}}$ denoting a field in the cross-section and $\mathbf{l}$ being given by $\mathbf{l} = [1, z, y]$ [13]. The, possibly nonlinear, constitutive relation yields the normal stress distribution, $\sigma_{\text{cs}} \equiv \sigma_{\text{cs}}[\epsilon_{\text{cs}}]$, whose resultants are the internal forces in each section

$$\mathbf{X}_s \equiv \mathbf{X}[\boldsymbol{\epsilon}_s] = \int_A \mathbf{l}^{\mathsf{T}} \sigma_{\text{cs}} \mathrm{d}A \tag{5}$$

square brackets $[\cdot]$ denoting a functional dependency. For the Euler-Bernoulli torsionless spatial element $\mathbf{X}_s = [N, M_y, M_z]^{\mathsf{T}}$. Because of the material nonlinearity, the "shapes" of the internal force fields $\mathbf{X}_s$ and of the generalized strain fields $\boldsymbol{\epsilon}_s$ are linearly incompatible. The cause to such incompatibility is a component of $\boldsymbol{\epsilon}_s$ which shall be denoted $\boldsymbol{\epsilon}_{\text{NL,free,s}}$ and that is such that the linear relation between the internal forces and $\boldsymbol{\epsilon}_s - \boldsymbol{\epsilon}_{\text{NL,free,s}}$ is given by $\mathbf{K}_{\text{aux,s}}$,

$$\mathbf{X}_s = \mathbf{K}_{\text{aux,s}}\left(\boldsymbol{\epsilon}_s - \boldsymbol{\epsilon}_{\text{NL,free,s}}\right) \iff \boldsymbol{\epsilon}_{\text{NL,free,s}} = \boldsymbol{\epsilon}_s - \mathbf{K}_{\text{aux,s}}^{-1}\mathbf{X}_s \tag{6}$$

see Fig. 2. This expression justifies the assumption of constant $\mathbf{K}_{\text{aux,s}}$ fields along each finite element. In general, the fields $\boldsymbol{\epsilon}_{\text{NL,free,s}}$ are incompatible with vanishing nodal displacements. The nonlinear component $\boldsymbol{\epsilon}_{\text{NL},0,s}$ of the generalized strain is therefore given by the sum of this irregular component $\boldsymbol{\epsilon}_{\text{NL,free,s}}$ with another one, corresponding to a statically admissible field, $\mathbf{b}_s\mathbf{p}_e$, where $\mathbf{p}_e$ are the n_X hyperstatic forces required for null nodal displacements,

$$\boldsymbol{\epsilon}_{\text{NL},0,s} = \boldsymbol{\epsilon}_{\text{NL,free,s}} + \mathbf{K}_{\text{aux,s}}^{-1}\mathbf{b}_s\mathbf{p}_e \tag{7}$$

Inserting this decomposition into the PVF compatibility statement (1), yields

$$\int_0^L \mathbf{b}_s^{\mathsf{T}}\boldsymbol{\epsilon}_{\text{NL,free,s}}\mathrm{d}x + \left(\int_0^L \mathbf{b}_s^{\mathsf{T}}\mathbf{K}_{\text{aux,s}}^{-1}\mathbf{b}_s\right)\mathbf{p}_e\mathrm{d}x = \mathbf{0} \tag{8}$$

Introducing as intermediate quantities the n_X element discontinuities,

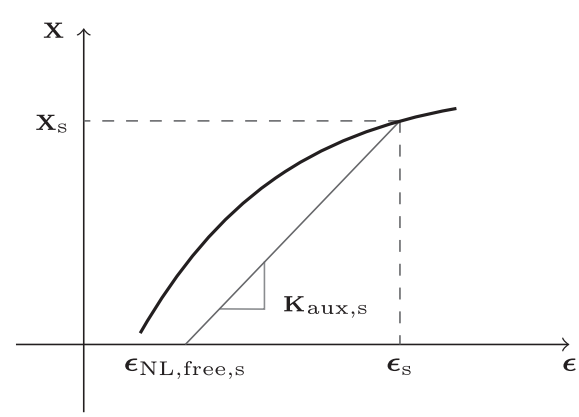

Fig. 2. Schematic illustration of expression (6).