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Topology optimization of geometrically nonlinear trusses with spurious eigenmodes control

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ABSTRACT

In this paper, topology optimization of geometrically nonlinear trusses with and without stability constraints is investigated. It is shown that if classical minimum compliance formulation is considered without any stability constraints, the optimized designs are unstable and convergence issues may be encountered in the nonlinear structural analyses. To address these issues, a minimum compliance formulation with critical load factor constraint is proposed together with a strategy based on spurious modal energy ratio to determine the true critical eigenmodes and the corresponding critical load factor. Several numerical examples are presented to demonstrate the effectiveness of the proposed approach, which show that the optimized truss topologies obtained using the proposed approach are stable. More importantly, the critical load constraint is able to guarantee that the first critical load of the optimized design is always above the applied load so that the proposed approach is free from convergence issues during the Newton Raphson solution process.

1. Introduction

Topology optimization of trusses is an important topic in structural optimization as it provides an efficient and flexible design technique [1–5]. Unlike classic sizing or shape optimization, topology optimization seeks the best layout of members by optimizing the given objective function while satisfying the prescribed constraints and boundary conditions [6,7]. The truss topology optimization problem is usually formulated and implemented in the framework of a ground structure approach [8]. In this approach, an interconnected initial mesh, termed "ground structure", is first generated, and then the inefficient members are subsequently removed during the optimization process. Small deformation and elastic material assumption is usually made in the truss topology optimization, which results in a geometrically linear truss model. In this case, the most commonly used optimization formulation is minimum compliance, subjected to the equilibrium equations and volume constraints, which is also called the stiffness design formulation [9–12]. This displacement based non-convex formulation can be either solved by various nonlinear programming (NLP) methods [7,13], or transformed into convex formulations which are solved by tailored optimization algorithms [12]. The stiffness design formulation can be also approximated in terms of member forces to yield a linear programming (LP) problem, which can be solved very efficiently with a large number of design variables [14–16].

As stability considerations are absent from the stiffness design formulations, the optimized topologies may be unstable. For topology optimization, a number of studies in the past have addressed the local and global stability issues in linear elastic truss topology optimization [17]. In most cases, the local stability of members is enforced by including constraints based on Euler buckling [18–20]. While the local stability issue can be addressed in the stiffness design formulations by including additional constraints based on the Euler buckling criterion, the final optimized topologies can still be globally unstable, as in the common case of a chain of collinearly connected truss members [18]. Although these collinear members can be merged into a one longer member through the node cancellation post-processing, it is shown that the long members also increase the potential for local instabilities, and the global stability may still not be ensured [19]. To address this issue, linear truss topology optimization considering global stability has been investigated in the past. For instance, Ben-Tal et al. [17] and Kočvara [21] included a linear global stability constraint into the compliance formulation. Guo et al. [22] incorporated overlapping bars in the ground structure aiming to address the difficulty caused by hinge cancellation pointed out by Rozvany [18].

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These aforementioned studies are mostly based on stiffness design formulation and are solved using NLP, since the LP formulations cannot be recovered in the case where stability constraints are included. Moreover, all the studies mentioned above are restricted to linear elastic truss models, and the so-called linear buckling analysis is adopted to evaluate the critical loads. The linear buckling analysis is based on the assumption that geometric nonlinearities are insignificant and this assumption can lead to incorrect estimation of the critical loads if the actual deformations are large [23]. Therefore, it is necessary to employ geometrically nonlinear models in the stability analysis for accurately determining the critical loads. However, this will lead to a nonlinear system, and the equilibrium solution of such a system has to be characterized with respect to critical points including bifurcation and limit points. Such nonlinear analysis is also rather meaningful, since it will naturally capture the nonlinear behavior of truss systems under large deformations. Thus topology optimization of geometrically nonlinear trusses is the focus of this paper. It should be noted that only the literature considering truss topology optimization is reviewed above; details about nonlinear topology optimization in continuum settings can be found in Refs. [24,25] and references therein. In addition, sizing optimization of geometrical nonlinear trusses is not reviewed here and the details can be found elsewhere [26–29].

For geometrically nonlinear trusses, Ramos and Paulino [30] recently proposed a convex topology optimization formulation using a potential energy approach. However, stability issues were not addressed in that study. In this study, topology optimization of geometrically nonlinear trusses is carried out using optimization formulations with and without critical load constraints. To this end, a new optimization formulation is proposed for nonlinear truss topology optimization, wherein the global stability constraint is explicitly incorporated using the minimum critical load associated with the true eigenmode. To address the spurious modes issue associated with the low density members [31–33], a new spurious mode identification approach is presented based on the spurious modal energy ratio. Various test cases are presented to demonstrate the effectiveness of the proposed approaches for handling stability issues in geometrically nonlinear trusses.

The paper is organized as follows: In Section 2, the large deformation kinematics/kinetics of geometrically nonlinear truss, the equilibrium equations and solution techniques are presented. Section 3 presents the critical load factor estimation based on the nonlinear analysis. In Section 4, the topology optimization formulations are stated and the relevant sensitivity analyses are carried out. Section 5 describes the approach to identify the localized eigenmodes using spurious modal strain energy ratio. Various test cases are presented in Section 6 to demonstrate the effectiveness of the proposed approaches. Finally, the important remarks and conclusions are given in Section 7.

2. Geometrically nonlinear truss analysis

In this section, a geometrically nonlinear hyperelastic truss model based on Green-Lagrangian strain is briefly described in which the large deformation kinematics are taken into consideration.

2.1. Kinematics and kinetics

Consider a truss member AB in initial configuration with initial length L_0, area A_0 and volume $V_0 = L_0A_0$ as shown in Fig. 1. After deformation $\boldsymbol{u}_a$ and $\boldsymbol{u}_b$ of nodes A and B, respectively, the current configuration for the member becomes ab with member length L. The member length before and after the deformation are given by

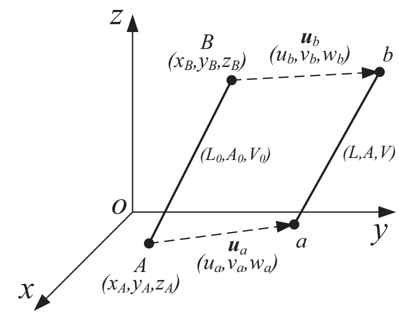

Fig. 1. Kinematics of a truss member.

$$L_0 = \sqrt{\boldsymbol{X}_{BA}^T\boldsymbol{X}_{BA}} \tag{1}$$

$$L = \sqrt{(\boldsymbol{X}_{BA} + \boldsymbol{u}_{ba})^T(\boldsymbol{X}_{BA} + \boldsymbol{u}_{ba})} \tag{2}$$

where the coordinate difference vector $\boldsymbol{X}_{BA}$ and the relative displacement vector $\boldsymbol{u}_{ba}$ between nodes A and B are given as

$$\boldsymbol{X}_{BA} = [X_{BA}, Y_{BA}, Z_{BA}]^T = [X_B - X_A, Y_B - Y_A, Z_B - Z_A]^T \tag{3}$$

$$\boldsymbol{u}_{ba} = [u_{ba}, v_{ba}, w_{ba}]^T = [u_b - u_a, v_b - v_a, w_b - w_a]^T \tag{4}$$

where (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B) are the nodal coordinates in the initial configuration of member nodes A and B, while (u_a, v_a, w_a) and (u_b, v_b, w_b) are the displacements of the nodes A and B, respectively, as shown in Fig. 1. The Green-Lagrangian strain is used in this study which is given by

$$\varepsilon = \frac{1}{2}(\lambda^2 - 1) \tag{5}$$

where $\lambda = L/L_0$ is the stretch ratio for the truss member. Note that the other strain measures such as logarithmic strain and Euler-Almansi strain [34], can also be employed to formulate the finite deformation truss model [30]. Only the elastic material behavior is considered and, in this case, the stress is given by

$$\sigma = E\varepsilon \tag{6}$$

2.2. Equilibrium by principal of virtual work

The internal (W_{int}) and external (W_{ext}) virtual work for the truss structure can be expressed as

$$\begin{aligned} W_{int} &= \sum_{e=1}^{n_{el}} W_{int}^e = \delta\boldsymbol{u}^T\boldsymbol{F}_{int}(\boldsymbol{u}) \text{ where} \\ \boldsymbol{F}_{int}(\boldsymbol{u}) &= \mathscr{A}_{e=1}^{n_{el}}\boldsymbol{F}_{int}^e = \mathscr{A}_{e=1}^{n_{el}}(\boldsymbol{B}^{e^T}A_0^eL_0^e\sigma^e) \end{aligned} \tag{7}$$

$$W_{ext} = \delta\boldsymbol{u}^T\boldsymbol{P} \tag{8}$$

where $\mathscr{A}$ is the standard finite element assembly operator, n_{el} denotes the total number of truss members, $\boldsymbol{P}$ is the external nodal force vector and $\delta\boldsymbol{u}$ is the global virtual displacement vector. Also, the displacement-gradient operator $\boldsymbol{B}^e$ for the eth member is given by

$$\begin{aligned} \boldsymbol{B}^e = \frac{1}{L_0^{e2}}[&-X_{BA}^e - u_{ba}^e, -Y_{BA}^e - v_{ba}^e, -Z_{BA}^e - w_{ba}^e, X_{BA}^e + u_{ba}^e, Y_{BA}^e \\ &+ v_{ba}^e, Z_{BA}^e + w_{ba}^e] \end{aligned} \tag{9}$$

The principle of virtual work implies that $W_{int} = W_{ext}\forall\delta\boldsymbol{u}$, which after the application of boundary conditions yields the equilibrium equations in residual form as