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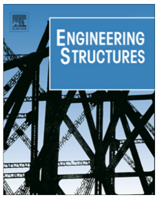

Instantaneous deflection calculation for steel fibre reinforced concrete one way members

Ali Amin[a,*], Stephen J. Foster[b], Walter Kaufmann[c]

[a] Centre for Infrastructure Engineering and Safety, School of Civil and Environmental Engineering, The University of New South Wales, Australia
[b] Head of School, Civil and Environmental Engineering, The University of New South Wales, Australia
[c] Institute of Structural Engineering, Swiss Federal Institute of Technology (ETH), Switzerland

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ABSTRACT

In this paper, a simple yet accurate method for determining the instantaneous deflection of steel fibre reinforced concrete (SFRC) one way members is presented. The model builds upon previous work conducted by the authors by considering the beneficial effect of the fibres across a crack to the tension stiffening relationship of SFRC. In turn, the increased stiffness across the crack is translated into curvature and then to deflection. The model reported herein is founded upon a previously reported adaptation of the Tension Chord Model of Marti et al. (1998) for the prediction of deflections for RC beams and is shown to provide good correlations to test data.

1. Introduction

The addition of fibres to concrete has a history in academia of over fifty years [1] and while the majority of these studies have focussed on the behaviour of members containing steel fibres only, in many, or indeed most, practical applications of steel fibre reinforced concrete (SFRC) construction, structural members are co-reinforced with fibres and conventional reinforcing steel [2–5]. By adding fibres to concrete, the primary objective is to bridge cracks once they form and hence provide some post cracking resistance in tension [6,7]. Prior to cracking, the contribution of the fibres to the tensile strength of concrete is, in most part, negligible; for a fibre to be engaged, a small opening at the crack must occur in order to activate the fibre matrix bond which is mostly developed through mechanical means (i.e. fibre anchorage and/or fibre snubbing) [8–10]. When both forms of reinforcement bridge a crack, the stress transmitted across the crack is shared between the reinforcing bar and the fibres [11]. Consequently, the average strain in the conventional steel bar is lower than it would be without the fibres between the cracks, and this leads to finer and more closely spaced cracks [12,13].

When examining the response of softening SFRC at the material level, the post cracking strength of the SFRC is localised at a single discrete crack. However, in analyses that require models for cracked concrete that use average stresses and strains to predict member behaviour, such as in determining deformation, it is necessary to consider tension stiffening effects. Consider the tensile stress-strain response of a plain concrete tension tie member, square in cross section and reinforced with a single concentric reinforcing bar (Fig. 1). Prior to cracking, the member behaves elastically. The slope of the σ-ε relationship is directly proportional to the transformed sectional area. On first cracking of the tension tie (f_{cr}), the stiffness of the member drops significantly; the member, however, is uncracked at other sections. As load increases, more cracks develop along the length of the tension tie and the average stiffness of the member further decreases. If the concrete between the cracks carried no tensile stress, then the response would asymptotically approach that of the bare bar. The difference between this behaviour and that shown in Fig. 1 is referred to as 'tension stiffening' and is attributed to the ability of concrete to carry tension between cracks as a result of the bond which is developed between the reinforcing steel and concrete. This, in turn, increases the member's rigidity/stiffness prior to the yielding of the reinforcement and hence, affects the deformation and cracking characteristics of the member.

The tension stiffening behaviour of SFRC is considerably different to that of plain concrete, once the member cracks. While plain concrete tension ties can only carry tension between the cracks, even at small crack widths, SFRC is able to transmit tensile stresses across cracks in addition to between them. This leads to a greater resistance in tension, not just after cracking, but also after yielding

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* Corresponding author.
E-mail address: Ali.Amin@unsw.edu.au (A. Amin).

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Notation

A_F	cumulative area of fibres crossing crack
A_f	cross-sectional area of single fibre
A_{sc}	area of compressive longitudinal steel reinforcement
A_{st}	area of tensile longitudinal steel reinforcement
a	shear span
b	width of member
D	depth of member
d	effective depth
d_f	diameter of fibre
d_n	depth of neutral axis
d_{sc}	depth to compressive longitudinal reinforcement
E_c	elastic modulus of concrete
E_s	elastic modulus of steel reinforcement
$f_{0.5}$	residual tensile strength calculated at a COD = 0.5 mm
f_{cm}	mean compressive strength of concrete
f_{cr}	cracking stress of tension tie
f_{ct}	tensile strength of concrete matrix
f_{sy}	yield strength of longitudinal reinforcement
I_{cr}	cracked second moment of area
I_e	effective second moment of area
I_g	gross second moment of area
k_t	orientation factor
l	span length
l_f	length of fibre
M_a	applied moment
M_{cr}	cracking moment
n	modular ratio
P	applied load
s_r	distance between cracks
w	crack width (COD); uniform distributed load
$\Delta\delta_0$	stiffening effect on uncracked concrete
$\Delta\delta_1$	stiffening effect on cracked concrete
δ	midspan deflection
δ_1	midspan deflection assuming fully cracked section
ε	strain
ε_0	strain at extreme compressive fibre
ε_{sc}	longitudinal compressive reinforcing steel strain
ε_{st}	longitudinal tensile reinforcing steel strain
ε_{sy}	yield strain in the longitudinal reinforcement
λ	a factor
σ	stress
σ_c	maximum stress in concrete matrix
$\sigma_{c,avg}$	average tensile stress provided concrete and steel fibres
$\sigma_{cr,\mathrm{exp}}$	experimental cracking strength of matrix
σ_f	tensile stress carried by steel fibres
$\sigma_{S,avg}$	average tensile stress in longitudinal steel
σ_{sc}	stress in compressive longitudinal reinforcement
$\sigma_{S,cr}$	tensile stress in longitudinal steel at crack
σ_{st}	stress in tensile longitudinal reinforcement
ρ	tensile longitudinal reinforcing ratio
ρ'	compressive longitudinal reinforcing ratio
ρ_f	supplied steel fibre volumetric ratio
ρ_t	effective reinforcement ratio
τ_b	bond stress between fibres and concrete matrix
χ	curvature
ξ	uncracked length parameter

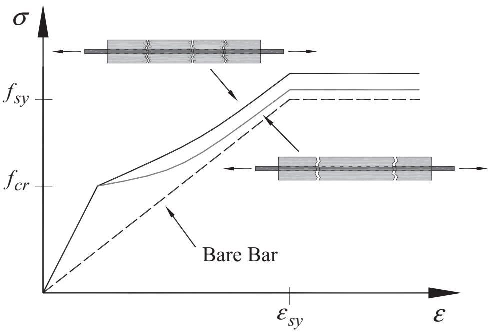

Fig. 1. Tension stiffening of SFRC and plain concrete tension ties.

(f_{sy}) of the reinforcing bar (as the fibres can still be effective at this stage). A detailed description of tension stiffening pertaining to SFRC can be found in [13–16]. With the quantification of tension stiffening and crack spacing, the determination of deflections of SFRC one way members can be realised. To this end, the tension stiffening relationship for SFRC tension ties previously developed by [13] is extended to determine the instantaneous deflections of SFRC beams within service loading.

2. Tension stiffening model for SFRC

Amin et al. (2016) [13] developed a generalised tension stiffening model for SFRC founded upon the Tension Chord Model (TCM) of [17] and reported in [2]. The TCM is a physically consistent tool used to model the load-deformation behaviour of tensile members with uniaxial stress states, such as in tension ties and tension chords in one way bending members. In their model, [13] considered the stress along a given tension tie and assumed that the resistance provided by the three components of the member (concrete bond, steel fibres and steel reinforcing bar) could be determined independently. This is shown in Fig. 2, with the main difference to the TCM being that at the crack, the fibres carry a stress equivalent to $\sigma_f(w)$. By compatibility, midway between cracks, the stresses in the reinforcement are a minimum and conversely, the stresses in the concrete reach their maximum value, f_{ct}, at the onset of cracking.

In developing an expression for the average tension stiffening along a SFRC tension tie, [13] considered the stress distributions that occur at the two extreme scenarios of crack spacing. The element shown in Fig. 3a illustrates the maximum crack spacing (s_{r0}) for a SFRC tension tie. This occurs when the sections midway between two primary cracks are at the onset of cracking. Conversely, the minimum crack spacing ($s_{r0}/2$) scenario occurs when these sections crack to form new primary cracks (see Fig. 3b). No further primary cracking can occur if the distance between the cracks is not large enough to develop sufficient force (through bond) to allow the stress within the concrete to reach its tensile strength, f_{ct}, and the maximum force in the concrete, in this hypothetical case, is limited to $f_{ct}/2$.

For the maximum crack spacing scenario presented in Fig. 3a, the average stress carried by the fibres and concrete is equivalent to:

$$\sigma_{C,avg,\max} = 0.5 f_{ct}\left(1 + \frac{\sigma_f(w)}{f_{ct}}\right) \qquad (1)$$

where $\sigma_f(w)$ is the residual tensile stress carried by the SFRC at a crack opening displacement (COD), w.

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