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Bayesian-network-based system identification of spatial distribution of structural parameters

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ABSTRACT

Structural deterioration or damage in civil infrastructures may results in severe losses of properties and human lives. To prevent such situations, structure health monitoring (SHM) technologies have been developed in recent decades. As one of the SHM technologies, so-called system identification (SI) methods aim to estimate structural parameters by minimizing an error function consisting of measured and calculated responses of a structure under the same loading condition. Such optimization-based SI algorithms may suffer from ill-posedness of the inverse problem, which may result in non-uniqueness of solutions or non-stability of the optimization process. In this paper, in order to avoid issues related to ill-posedness, an SI method based on the Bayesian Network (BN) technology is developed, especially for probabilistic identification of spatial distribution of structural parameters. Utilizing graph theories, a BN describes random variables by nodes connected by links, which represent conditional probability tables (CPT) explaining the probabilistic relationship between the linked nodes. For effective SI using BN, this study proposes a BN graph model, which employs a bi-variate Gaussian function as a shape function describing the spatial distribution of a structural parameter with a small number of parameters. The parameters of the Gaussian (shape) function are considered as nodes in the BN to describe various spatial distribution patterns using a small number of parent nodes. Using this modeling approach, the number of the BN nodes is not affected by the size of the finite element (FE) mesh. Thus, the approach prevents the BN graph from growing to an intractable size even if the FE analysis requires smaller meshes to improve the accuracy of the structural analysis. Using the constructed BN, information on applied loads, and observed structural responses, a BN inference algorithm effectively updates the prior distribution of spatial distribution parameters to the posterior distribution. The proposed method is tested and demonstrated by numerical examples. Using a variety of structural deterioration scenarios, the SI results by the proposed method are compared with those by maximum likelihood estimation (MLE), and an FE-updating method. During the test, the influences of measurement errors, and incomplete/missing data on the performance of the methods are also investigated. The results show that the proposed SI approach is more stable and robust than the other tested methods, and potentially has more merits that are worth investigating in future research.

1. Introduction

Maintaining the healthy conditions of civil infrastructure systems is important for preventing the losses of properties and lives. Throughout its life-cycle, a structure is subject to deterioration, aging or fault of material, which may weaken the structure to an uncertain level. To avoid catastrophic events caused by such structural weakness, it is essential to monitor the structural health of civil infrastructures in construction or use. For this reason, structure health monitoring (SHM) has been exploited to collect data from sensors on structures and estimate the actual values of structural parameters changed due to deterioration or damage of structures in use [1–6]. As one of the main SHM approaches, so-called system identification (SI) methods, which generally refers to non-destructive inverse analysis to estimate structural or modal parameters based on the measured responses of a structure subjected to loads, were developed in past decades. For example, Siringoringo and Fujino [7] proposed an SI method using ambient vibration response of a suspension bridge. In general, SI problems are solved by minimizing an error function consisting of measured and calculated responses [8–10]. Due to the nature of such optimization problems and noises of measured data, many SI algorithms have suffered

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from so-called "ill-posedness," which often refers to non-uniqueness and non-stability of solution in inverse problems [11–13]. There are various case studies and applications of inverse problems, and numerous techniques have been developed to improve accuracy, stability, and efficiency [14–17].

As an effort to overcome such challenges, this paper explores an idea of applying the Bayesian network (BN) technology to SI problems. BN is a probabilistic graphical model in which random variables are represented by nodes and their probabilistic dependencies are expressed by links [18,19]. These links represent conditional probabilistic tables (CPT), which contain the conditional probability distribution of "child" nodes given possible outcomes of their "parent" nodes. Once actual outcomes of random variables in a designed BN become available from measurement or other sources, the probability distributions of all nodes in the BN are readily updated to the posterior distributions using efficient BN inference algorithms. The BN technology has been recently applied to a wide range of areas including computer science, diagnosis algorithms, decision support systems, and social sciences because of the following merits [20,21]: (1) a BN provides a graphical, powerful, and efficient tool for describing complex systems consisting of interacting uncertain components; (2) a BN allows for efficient probabilistic updating and assessment of component/system performance; and (3) BN's graphical representations of uncertainties enable engineers or decision-makers to understand the interdependencies between random variables intuitively, and visually identify critical nodes after the BN is updated by new information called "evidence." Recently, making use of these merits, Straub [22] assessed the risk caused by rockfall hazard using BN. The BN technology has been applied to other natural hazards as well, e.g., typhoon [23], avalanches [24], and multiple hazards on infrastructure system [21,25] for decision-making based on the quantified risk. In addition, Richard et al. [26] developed a methodology employing reliability method and BN for robust updating of nonlinear structural models.

There are several BN applications to structures for the purpose of robust updating [26] and prediction of remaining strength [27]. In these studies, point estimates are made for specific parameters. By contrast, this paper, as the first attempt of BN-based SI, focuses on probabilistic identification of the spatial distribution of structural parameters by developing an effective BN modeling framework. For this purpose, parameters of spatial distribution model of the structural parameter of interest and structural responses are represented by nodes in BN, and CPTs are constructed based on Monte Carlo simulations (MCS) of forward structural analysis [28]. Then, using BN inference algorithms, posterior distributions of structural parameters given measured responses are obtained. This approach allows us to identify spatial distribution of structural parameters without using an optimization process usually required for inverse analysis, which is prone to issues caused by the ill-posedness. If a BN uses nodes representing parameters at particular locations, as a mesh size in finite element (FE) analysis [28,29] decreases, the number of the nodes in BN and computational costs of inference quickly increases. To address this issue, a bivariate Gaussian function is introduced to describe the spatial distribution of material parameters such as Young's modulus. As a result, the number of nodes representing the spatial distribution of the material parameter is fixed to six regardless of the mesh size.

The proposed approach is demonstrated and tested by numerical examples. The results are compared with those by the maximum likelihood estimation (MLE) and one of the existing SI methods, so-called FE-updating. For each of the assumed spatial distribution scenarios, relative errors are introduced to the measured responses at 1%, 5%, 10%, and 15% levels to simulate noises in the measurement. Through many scenarios and assumed error levels, the stability and robustness of the proposed BN-based SI are tested and compared with the other methods. In addition, the proposed approach is applied to cases in which multiple sets of measurements are available. In such cases, the posterior distributions from one set can be used as prior distributions for the Bayesian inference using the next set. The performance of this approach is also investigated by the numerical examples.

After providing theoretical backgrounds on SI, Bayesian parameter estimation, and Bayesian Network (Section 2), the paper proposes a new SI method based on Bayesian Network in Section 3. Then, the proposed method is tested and demonstrated by numerical examples in Section 4. Section 5 summarizes the paper and lists topics of future studies.

2. Theoretical backgrounds

2.1. System identification for linear elastic continua

Most of the existing SI algorithms are based on the minimization of the error between calculated responses (based on assumed values of structural parameters) and measured responses [8–10]. When the assumed structural model describing the mechanical system does not represent the actual structure well enough to overcome noises in the measured responses, the numerical solution of the optimization problem can be highly unstable. Therefore, a proper structural modeling is critical for SI. In most cases, FE models are used to construct the stiffness matrix in the equilibrium equation. For a linear elastic continuum, the static equilibrium is described as

$$\mathbf{K}(\mathbf{x})\mathbf{U} = \mathbf{P} \tag{1}$$

where $\mathbf{K}$, $\mathbf{U}$, $\mathbf{P}$, and $\mathbf{x}$ respectively denote the stiffness matrix, nodal displacement vector, nodal load vector, and structural parameters affecting the stiffness matrix such as material properties. With a change in $\mathbf{x}$, one could mathematically describe deterioration of the material, an inclusion of disparate materials, or crack decreasing the elastic properties around damage.

Then, an error function to be minimized is defined by using the displacement vector in Eq. (1), i.e.

$$\Pi_E = \frac{1}{2}\|\mathbf{f}(\mathbf{x}, \mathbf{U}^c, \mathbf{U}^m)\|^2 \quad \text{subject to } \mathbf{R}(\mathbf{x}) \leqslant 0 \tag{2}$$

where Π_E denotes the error function, and $\|\cdot\|$ represents the Euclidean norm of the error function vector $\mathbf{f}$. In addition, $\mathbf{U}^c$, $\mathbf{U}^m$, and $\mathbf{R}(\mathbf{x})$ are vectors of calculated displacements, measured displacements, and functions describing the constraints for the structural parameters, respectively. The vector $\mathbf{f}$ in the Euclidean norm quantifies the errors of the structural model for the given value of $\mathbf{x}$. The following two types of error functions are often used for SI [30,31]:

$$\mathbf{f}(\mathbf{x}, \mathbf{U}^c, \mathbf{U}^m) = \mathbf{U}^c(\mathbf{x}) - \mathbf{U}^m \tag{3}$$

$$\mathbf{f}(\mathbf{x}, \mathbf{U}^c, \mathbf{U}^m) = \mathbf{f}(\mathbf{x}, \mathbf{U}^m) = \mathbf{K}(\mathbf{x})\mathbf{U}^m - \mathbf{P} \tag{4}$$

Eq. (3), termed as a nonlinear least squares problem [12,29], quantifies the error in terms of the displacement. Since the error term involves the calculated response $\mathbf{U}^c(\mathbf{x})$, the optimization requires performing FE analysis [28], i.e. solving Eq. (1), iteratively during the optimization. Although this approach does not need full measurement, it might suffer from issues related to convergence, and is time-consuming in general. On the other hand, using the error definition in Eq. (4), termed as a linear least squares problem [11], the optimization does not require solving Eq. (1) iteratively. In this linear approach, the optimization problem is solved based on the derivative of the error function defined using L_2-norm. However, a full measurement is needed to avoid rank-deficiency in the equilibrium equation. As Park [16] showed, SI using Eq. (4), often termed as equation error estimator (EEE) with L_2-norm, may expe-