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Modeling of the structural behavior of multilayer laminated glass beams: Flexural and torsional stiffness and lateral-torsional buckling

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A B S T R A C T

In recent years several investigations were performed about the behavior of laminated structural glass elements, namely in terms of their flexural and torsional stiffness, with the lateral-torsional buckling of beams being one of the most relevant and complex topics. Various analytical formulations were proposed to describe the equivalent stiffness of laminated elements; however, none covers more than three layers of glass in a comprehensive and unified manner, and those that exist are not consensual. This work proposes a new formulation, based on sandwich theory, which provides equivalent results to previous formulations in a limited set of conditions, but that is also able to characterize the behavior of simply supported laminated glass columns and beams up to five layers, subjected to compressive axial loads, mid-span loads, uniformly distributed loads, four-point bending, pure bending or torsion. The fundamentals of the formulation presented in this paper allow it to be extended to a larger number of layers and to different load and support conditions. The proposed formulation is assessed by means of a parametric study based on the comparison with numerical results retrieved from finite element simulations, in order to assess the range of validity of each expression. Two analytical approaches for the lateral-torsional buckling problem are studied in detail, with their fundamentals being explained. Another formulation, proposed in Australian Standard AS 1288, is also addressed. An experimental assessment of the work developed is achieved by comparing the results from flexural tests available in the literature with analytical and numerical predictions.

1. Introduction

In light of current architectural trends, glass is being increasingly used in several non-structural and structural applications requiring transparent solutions.

Glass beams or glass-fins can be part of fully glazed solutions, resisting wind loads acting on a facade or supporting floors, roofs or stairways. The need for redundancy requires these elements to be laminated, *i.e.*, to be composed of multiple glass layers bonded by an interlayer. Because of their high slenderness, glass-fins are susceptible to lateral-torsional buckling.

In recent years, several researchers have studied the lateral-torsional buckling phenomenon in laminated glass beams, at both small (*e.g.*, [1]) and large (*e.g.*, [2]) scales. Experimental, analytical and numerical studies have been conducted in order to better understand, for example, the structural behavior of laminated glass beams [3–7], the influence of the visco-elastic properties of multiple interlayer products [8,9], the influence of different geometrical imperfections [10] and glass fracture mechanics [11,12]. There is still much work to be done on this subject, as attested by the variety of analytical formulations that have been put forward to define the same engineering problems. Indeed, there is not yet a unified and commonly adopted formulation to assess this phenomenon. Additionally, there is also lack of generality on the proposed formulations and their field of application does not cover many practical situations.

The work presented here aims at (i) extending and consolidating the analytical formulations proposed to compute the equivalent flexural and torsional stiffness, so that they can be applied to laminated glass elements with a larger number of layers (more than three) and other loading conditions, and (ii) assessing various lateral-torsional buckling formulations, in order to support their application.

On the one hand, among the available analytical expressions, this work aims at verifying which ones are valid and more accurate, so that they can be extended to laminated glass elements with more than three layers. The work also aims at determining the range of validity of all the expressions analyzed in this work. The assessment of the validity and accuracy of the available analytical

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Nomenclature

$\alpha, \beta, \theta, \phi, \psi, \lambda_f, \xi_f$	equivalent flexural stiffness auxiliary parameter
δ	Dirac's function
γ/γ_0	transverse/warping shear strain
$\gamma_{ij},\ \gamma_{ij,k}$	shear strain
$\gamma_{int},\ \gamma_{int,i}$	shear strain in the interlayer
γ_{xs}	shear strain in a generically shaped cross-section
$\lambda_t,\ \lambda_{t,1},\ \lambda_{t,2},\ \mu,\ \rho,\ \gamma_1,\ \gamma_2,\ \xi_t$	torsional stiffness auxiliary parameter
ν, ν_{int}	glass/interlayer Poisson ratio
$\omega(s)$	sectorial area of a cross-section with respect to its shear center
ϕ_0	initial rotation
$\phi_{0,\max}$	initial rotation of the mid-span cross-section
$\phi_{\max}$	rotation of the mid-span cross-section
ψ_{ij}	shape function for the Galerkin's method
$\sigma/\sigma_{\max}$	maximum normal stress in the exterior layers, along the span/at mid-span
$\tau, \tau_{ij},\ \tau_{ij,k}$	shear stress
$\tau_{int},\ \tau_{int,i}$	shear stress in the interlayer
ξ_σ	equivalent second moment of area auxiliary parameter
$\bar{u}_i$	solution of the Galerkin's method
A	equivalent flexural stiffness auxiliary parameter; area
a	equivalent flexural stiffness auxiliary parameter
a_i	distance between the center-lines of adjacent glass layers
b	width
$C_1,\ C_2,\ g_2,\ g_3$	auxiliary parameter for the lateral-torsional behavior of a beam subjected to in-plane transverse loads
d	distance between the center-lines of the exterior glass layers
E/E_{int}	glass/interlayer elastic modulus
EI	flexural stiffness of a cross-section
EI_f	equivalent flexural stiffness of a laminated glass element
F	transverse concentrated load
G/G_{int}	glass/interlayer shear modulus
GJ_t	torsional stiffness of a laminated glass element
I_s	component of the second moment of area of the glass layers with respect to the neutral axis
I_σ	equivalent second moment of area of a laminated glass element
I_{gl}	component of the second moment of area of the glass layers with respect to their own centroidal axes
J_s	torsion constant component associated with the shear contribution of the interlayer
J_{gl}	torsion constant component associated with the thin-walled contribution of the glass layers
L	span
L_1	length of the overhangs
L_a	distance between the loads and the supports in a four-point bending configuration
M	bending moment
M_{cr}	critical buckling moment of a glass beam
P	axial compressive load
P_{cr}	critical buckling load of a glass column
q	uniformly distributed load
q_{ij}	parameter of the Galerkin's method
S	first moment of area
T	torsional moment
t_i/t_{int}	thickness of the glass layers/interlayer plies
T_s	torsional moment component associated with the shear contribution of the interlayer
t_{tot}	total thickness of laminated glass element
$u,\ u^A,\ u^B,\ \bar{u},\ \bar{u}^A,\ \bar{u}^B$	warping deformation
V	shear force
v_0	initial lateral deflection
V_1	pure bending component of the shear force
$v_{0,\max}$	initial lateral deflection of the mid-span cross-section of a beam subjected to in-plane transverse loads
$v_{\max}$	lateral deflection of the mid-span cross-section of a beam subjected to in-plane transverse loads
w_1/w_2	pure bending/pure shear component of the deflection of a glass column or of a beam subjected to out-of-plane transverse loads
z_g	distance between the applied load and the shear center

expressions is based on the results obtained from numerical finite element models implemented here for such specific purpose. The analytical study that aims to extend the formulations (so that they can be applied to laminated glass elements with more than three layers) is based on sandwich theory, both regarding flexural and torsional stiffnesses. The study addresses critical buckling loads of columns subjected to compressive axial loads, deflections and stresses of beams subjected to transverse loads or pure bending, and rotations of beams subjected to torsional moments.

On the other hand, the underlying fundamentals of two existing formulations regarding the lateral-torsional buckling phenomenon are investigated. Based on experimental studies available in the literature [2,13], analytical and numerical simulations are compared with the experimental results. This also allows to further validate the previously mentioned analytical expressions, applied in this case to flexural tests on beams prone to lateral-torsional buckling.

2. Review of previous analytical studies

2.1. Summary of the main studies conducted

Several analytical studies conducted by different authors resulted in a relatively large number of formulations, which aim to characterize the stiffness of laminated glass elements. In the literature there are five main formulations proposed to define the equivalent flexural stiffness of laminated glass elements [4,5,14–16] and three main formulations to define their torsional stiffness [4,5,17].

In the case of columns subjected to compressive axial loads, the critical buckling load (P_{cr}) may be determined with the following equivalent flexural stiffness formulations applied to Euler's equation: (i) Luible's expressions [5], based on the sandwich theory work of Stamm and Witte [18] (referred here as "1-A"); (ii) Amadio and Bedon's expressions [14], derived from the composite beam theory proposed by Newmark [19] ("2-A"); and (iii) the "Wölfel-Bennison approach" [15], first proposed by Wölfel [20] ("3-A"). These three formulations yield the exact same results for this loading case. In the case of beams subjected to out-of-plane mid-span loads, the maximum deflection ($w_{\max}$) may be determined with the following equivalent flexural stiffness formulations applied to the monolithic beam deflection equation: (i) Luible's expressions [5], based on the sandwich theory work of Stamm and Witte [18] ("1-B"); (ii) the expressions proposed Kasper et al. [4] and adopted in the recent guideline *Guidance for European Structural Design of Glass Components* [21] ("2-B"); and (iii) the "Enhanced effective thickness approach" ("3-B"), derived by