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Modeling the tensile steel reinforcement strain in RC-beams subjected to cycles of loading and unloading

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ABSTRACT

Tension stiffening affects the strain distribution along the tensile reinforcement in a cracked reinforced concrete beam and in the tensile concrete between cracks. It also affects the overall stiffness and hence the deflection of the beam. In this paper, the results of experiments on eleven reinforced concrete beams with reinforcement ratios between 0.56% and 0.88% are reported. The overall strain in the reinforcement and the load-deflection response under both monotonic loading and cycles of loading and unloading were measured for each beam. Based on the experimental results, a model of the effective strain in the reinforcement is presented and is used to assess the effective moment of inertia of reinforced concrete beams subjected to in-service monotonic loading. Measurements from the test beams were used to calibrate a model of the steel-concrete interface damage caused by cycles of loading and unloading. The comparisons between predicted and measured overall stiffness and load-deflection responses show the validity of the present model.

1. Introduction

To ensure the serviceability of reinforced concrete structures, deflection control is an important design objective. Excessive concrete cracking and excessive deformation are one of the most common causes of damage and result in large annual cost to the construction industry.

Reinforced concrete beams in service are usually cracked, as the tensile strength of concrete is low [1]. Under normal service conditions, the concrete between the primary cracks is able to continue to carry tensile stress, due to the transfer of forces from the tensile reinforcement to the concrete through bond. This phenomenon is known as *tension stiffening*, which affects the beam's stiffness and hence its deflection, especially for lightly reinforced concrete beams [2]. As a result, the tension stiffening effect must be accurately modeled to simulate the in-service behavior of reinforced concrete structures, particularly under repeated loading [3].

Most of existing models for the flexural reinforced concrete beams deal with monotonic loading. The smeared-crack model is a popular way to simulate the tension stiffening effect. In this approach, an average stress-strain relation is considered for the whole tension area to account for the "average" deformation response of a cracked member [4]. A modified constitutive relationship for the steel reinforcement [5,6] or an updated descending branch of the tensile stress-strain curve for concrete have been developed and implemented in finite element analyses [5,7–12]. In addition, the so-called microscopic models based on the bond-slip mechanism and discrete cracking have been proposed by Floegl and Mang [13], Gupta and Maestrini [14], and Choi and Cheung [15].

Alternatively, several empirical models have been widely accepted by engineers in design for the control of deflections, involving determination of the effective moment of inertia (I_e) for a cracked member under monotonic loading. Branson developed a well-known model [16], which was adopted by the ACI Building Code [17]. Branson's equation gives a weighted average of the uncracked and cracked stiffness of the reinforced concrete cross-section at any load level, but it has been shown to overestimate the effective stiffness of lightly reinforced concrete beams and slabs [2]. In comparison with Branson's model, Bischoff suggested a weighted average of the uncracked and cracked flexibility of reinforced concrete cross-sections [18,19]. Experiments carried out on reinforced slabs having reinforcement ratios ranging between 0.18% and 0.84% demonstrated that Bischoff's model is more accurate than Branson's model for lightly reinforced concrete members [2]. A statistical study that employed data from nine

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Nomenclature

A_s	area of the tensile reinforcement
$A_{tc,eff}$	effective area of the tensile concrete
D_{ccc}	scalar damage variable
E_c	elastic modules of concrete
E_s	elastic modules of steel
I	moment of inertia
I_a	average moment of inertia
I_{cr}	moment of inertia of the fully cracked section
I_e	effective moment of inertia
I_{uncr}	moment of inertia of the uncracked section
$I(x)$	moment of inertia at a point x from the crack within the transfer length
L_{elem}	average crack spacing
L_t	transfer length
M_a	applied moment
M_{cr}	cracking moment
M_y	yield moment
$M(x)$	bending moment along the transfer length
N_c	total number of cracks spacing in the pure bending region
a	thickness of the reinforcement layers in the cross-sections
b	width of the rectangle cross-section
d	effective depth of the tensile reinforcement
d_b	bar diameter
f_c	compressive strength of concrete
f_{cu}	compressive cube strength of concrete
f_{tc}	tensile strength of concrete
f_y	yield strength of reinforcement
$g(x)$	interpolation function between cracks
h	height of the rectangle cross-sections
l_0	length of the neutral axis in the measure region
l_1	length of the reinforcement in the measure region
l_2	length measured with LVDT in the measure region
$l_{s,max}$	maximum cracks spacing
$l_{s,min}$	minimum cracks spacing
n	modular ratio (E_s/E_c)
r	radius of curvature of the deflected centroidal axis
r_e	effective radius of curvature
x	distance from the cracks
y_{oa}	average depth of the neutral between cracks
y_{oc}	depth of the neutral axis of the fully cracked section
y_{onc}	depth of the neutral axis of the uncracked section
$y_o(x)$	neutral axis depth along the transfer length
z_c	lever-arms of the internal forces on the cracked cross-section
z_{nc}	lever-arms of the internal forces on the uncracked cross-section
$z(x)$	lever-arms of the internal forces between cracks
Δ_m	measured overall lengthening of the tensile reinforcement
α	interpolation shape coefficient for effective strains of reinforcement
γ	evolution parameter of D_{ccc}
δ	displacement from LVDT to the tensile reinforcement
ε_{ce}	effective strain of the top fiber compressive concrete
$\varepsilon_{ce,cr}$	effective strain of the top fiber compressive concrete at the moment of cracking
$\varepsilon_{ce,y}$	effective strain of the top fiber compressive concrete at the moment of yielding
ε_s^p	permanent residual strain of tensile reinforcement
ε_{sav}	average strain of tensile reinforcement
ε_{se}	effective strain of tensile reinforcement
$\varepsilon_{se,cr}$	effective strain of the reinforcement at the moment of cracking
$\varepsilon_{se,y}$	effective strain of the reinforcement at the moment of yielding
ε_{snc}	reinforcement strain at the uncracked section (where $x \geqslant L_t$)
$\varepsilon_s(x)$	reinforcement strain between cracks
ε_{s0}	reinforcement strain at the cracked section (where $x = 0$)
$\varepsilon_{tc}(x)$	tensile concrete strain between cracks
$\varepsilon_{ts,max}$	concrete strain at the level of tensile reinforcement section at $x = L_t$
θ	beam's rotation
χ_{exp}	measured curvature of the reinforced concrete beam
$\chi_{predicted}$	predicted curvature of the reinforced concrete beam
$\bar{\chi}$	average ratio of $\chi_{predicted}/\chi_{exp}$
$v_{perm,cr}$	irreversible deflection of the cracked reinforced concrete beam
ρ	reinforcement ratio $\rho = A_s/(bd)$
ρ_{min}	minimum reinforcement ratio
σ_{s0}	stress of the tensile reinforcement at the cracked section
$\sigma_{s,ccc}$	critical axial steel stress at the cracked location related to cover-controlled cracking
$\bar{\sigma}$	relative stress intensity of the tensile reinforcement at the cracked section
$\chi(x)$	curvature at a point x from the crack within the transfer length

experimental programs involving a total of 80 specimens showed a similar conclusion that the Branson's model overestimated the stiffness significantly for reinforcement ratios ranging between 0.4% and 0.8% [20]. In fact, the accuracy of deflection predictions made using all the techniques decreased significantly for beams with small reinforcement ratios [20]. In this paper, an alternative empirical model to Bischoff and Branson's models is proposed. Contrary to Bischoff and Branson's models, which are both based on weighted averages of uncracked and cracked moment of inertia, the model proposed is based on the local measurement and modeling of the steel reinforcement strains in the tensile zone.

Due to the tension stiffening effect, the distribution of the tensile strain along the reinforcement bars between the cracks is not uniform, but rises to a maximum at each crack and drops to a minimum mid-way between adjacent cracks. In some experimental studies, strain gauges glued to the bar surface have been used to measure steel strain [21]. However, this is not entirely satisfactory, as the bond properties of the reinforcement bars are affected by the strain gauges [22]. Alternatively, magnetic methods and fiber-optical strain measurements have been used to measure the distribution of strain in the reinforcing steel [22,23]. In this paper, instead of measuring the distribution of reinforcing steel strain, a simple apparatus was adopted to measure the overall steel reinforcement strain over a length of the cracked tension zone containing two or more primary cracks. The mid-span deflection of the test beam was also measured. In total, eleven reinforced concrete beams were tested. Using the experimental results, a model for the overall steel reinforcement strain, termed the effective steel strain, has been developed for monotonic loading. The moment of inertia can then be deduced by homogenization across the height of the reinforced concrete beam, thereby providing an alternative way to calculate the effective moment of inertia (I_e). The performance of the new model is compared to results from Bischoff's model results for different reinforcement ratio.