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Incremental model formulation of age-dependent concrete character and its application

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ABSTRACT

An incremental format for an age-dependent constitutive equation was derived to account for the persistent change in creep-causing stress. This derivation was achieved by expanding the total form of the constitutive equation by a first-order Taylor series, with respect to the stress, creep, and shrinkage strains, and the elastic modulus of concrete. The development of the creep strain was depicted using a two-way parallel creep curve that was derived to remedy the disadvantage of the rate-of-creep method. The resulting incremental constitutive equation was defined using three basic equations for the creep, shrinkage, and development of the elastic modulus. Laboratory experiments were carried out on unreinforced and reinforced cylindrical concrete test specimens. The performance of the creep model was identified by creep tests on cylindrical specimens with stepwise loads, and compared with the performance of the effective modulus method, the age-adjusted effective modulus method, the rate-of-creep method, and the step-by-step method. The long-term behaviors of an existing three-span prestressed continuous double-T beam were analyzed using the presented age-dependent constitutive equation, and the results were compared with those calculated by the other age-dependent constitutive equations that were based on the creep models of the effective modulus method, the age-adjusted effective modulus method, the rate-of-creep method, and the step-by-step method.

1. Introduction

When the strain due to creep and shrinkage in concrete is restrained by internal or external constraints, the resulting change in the internal stress state changes the initial condition of creep strain development, which is strongly dependent on the level of internal stress. The pressure-sensitive nature of creep leads to a circulating loop phenomenon in the age-dependent creep-inducing process. The time-dependent behavior of a concrete bridge structure [1–10] under its self-weight illustrates the circulating loop phenomenon well, in which creep strains develop in proportion to the initial flexural normal stresses along the depth of the girder. Once the creep and shrinkage strain is restrained by reinforcement or steel parts, the internal stress state changes because of the mechanical strain developed as a restraining effect. This paper presents a constitutive formulation to depict the restraining effect by inter-relating creep development under time-varying stress with the mechanical relationship between stress and strain, in a way that accounts for the mechanical strain development due to the restraining effect.

A constitutive model of the age-dependent concrete characteristics was formulated using a two-level formulation framework, with a material level for depicting the creep strain due to the stress history, and a mechanical level for modeling the stress change due to the time-dependent development of mechanical strain. Most of the research on creep models has been performed to model creep phenomena under various conditions for mix proportions, curing environments, loading ages, and geometrical shape and dimensions [11–23]. However, when focusing on the circulating-loop phenomenon occurring in a time-dependent analysis of concrete structures, a creep model has to depict the creep strain developed under the time history loadings applied at different ages. A step-by-step method (SSM) based on the principle of superposition is a simple and robust way to model the creep strain over a time-varying stress history. The formulation approximates the entire stress history with stepwise stress variations, which are called incremental stresses, applied at the end of small time intervals [21]. When the creep is modeled based on this approximation, the model characteristics are dependent on the creep curve, which can be used to define the creep strain in each time interval. Several methods have been presented to depict the creep curve: the

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rate-of-creep method [24], the SSM [21], the effective modulus concept [25], and the ageing coefficient concept [26]. The performance of various models was extensively studied by Gilbert and Ranzi [21] through a comparison of mathematical formulations and numerical applications. The performance of each model depended on the assumptions made when modeling the ageing effects of concrete loaded at different ages on the creep strain development. Another aspect of a creep model is its compatibility with an age-dependent constitutive model that accounts for the change in the creep-causing stress and mechanical strain, as well as the global equilibrium equations for imposing constraints on the creep and shrinkage strains. This compatibility requirement on the material, the mechanical (local), and the global levels requires a consistent formulation framework for a total or incremental formulation. This work adopts an incremental creep curve formulation that combines a vertical parallel creep curve concept for the rate-of-creep method [24], as well as a horizontal parallel creep curve concept, both of which are aligned with an incremental time-dependent formulation at the mechanical and global levels.

The missing link between the material and the mechanical level was resolved using a constitutive equation that accounts for the development of the mechanical strain due to the internal or external restraint imposed on the creep and shrinkage strains. The performance of the constitutive model is dependent on the nature of the underlying creep model used to depict the creep under a time-varying stress history. A few constitutive models have already been presented: Glanville's rate-of-creep method (RCM) [24], Faber's effective modulus method (EMM) [25], and Bažant's age-adjusted effective modulus method (AEMM) [26]. Of these constitutive models, the AEMM has been widely accepted because of its simple and robust representation. However, because the model was inherently developed to describe the creep behaviors at different loading ages in a total format, a disadvantage is encountered when using it when the finite element equilibrium equation for displacement analysis is formulated in an incremental framework to account for multistep load histories.

This paper presents an incremental format for an age-dependent constitutive equation to describe the persistent changes in creep-causing stress, as well as the shrinkage strain and the development of the elastic modulus. The constitutive formulation was achieved by expanding the total form of the constitutive equation by using a first-order Taylor series. The total form of the age-dependent stress vs. strain relationship was specified in this formulation for two phases: (1) a nonconstrained deforming phase due to creep and shrinkage and (2) a constrained (mechanical) deforming phase due to constraints on the creep and shrinkage strains. The resulting incremental constitutive equation requires three basic equations for creep under constant pressure, shrinkage, and the development of the elastic modulus, which are either experimentally determined or given by formulae specified by the American Concrete Institute (ACI) [11] or the Euro-International Concrete Committee [17].

Laboratory experiments were carried out on two types of cylindrical specimens: with and without axial reinforcements, where the case with axial reinforcements was designed to examine the restraining effects on the creep and shrinkage strains. The three basic equations for defining the age-dependent characteristics of the test concrete were obtained from the results of the tests on the unreinforced specimens. The overall performance and characteristics of the presented constitutive equation were examined through comparisons between the measured and the predicted results. Time-dependent finite beam element analyses were carried out for an existing prestressed concrete (PSC) continuous double-T beam with bonded prestressing tendons. The time-dependent behavior of this double-T beam was also analyzed using finite beam element formulations employing the other four constitutive formulations of the RCM, EMM, SSM, and AEMM. The performance of the presented constitutive equation was compared with the performances of the other four constitutive equations through the numerical predictions. The model parameter values, creep function, shrinkage function, and development of the elastic modulus were obtained from ACI Committee 209 [11].

2. Incremental creep formulation

A two-way parallel creep curve formulation was used to depict the creep strain under a time-varying stress history. The creep curve was derived by summing the two parallel creep curves in the vertical and parallel directions, where "parallel" indicates geometrical equality among the tangents of the creep curves corresponding to individual loadings at different ages. Fig. 1 illustrates the geometrical meaning of "parallel," where it becomes a vertical parallel creep curve when the tangents of the curves are assumed to be identical along a vertical line, and it becomes a horizontal curve when these tangents are assumed to be identical along a horizontal line. In Fig. 1, $J'(t,t_o)$ designates a creep function, and the relationship between the creep coefficient $\varphi(t,t_o)$ and the creep compliance function $J(t,t_o)$ under a single stress history $\sigma(t_o)$ applied at time t_o is expressed as follows:

$$\begin{aligned}\varepsilon(t) &= \frac{1+\varphi(t,t_o)}{E_c(t_o)}\sigma_c(t_o) = \left\{\frac{1}{E_c(t_o)} + J'(t,t_o)\right\}\sigma_c(t_o) \\ &= J(t,t_o)\sigma_c(t_o) \qquad (1)\end{aligned}$$

Based on the vertical parallel creep curve assumption in the RCM [24], the creep strain $\varepsilon_{cr}(t_n)$ at time t_n due to loadings at the ages of $t_o, t_1, t_2, t_3, \ldots, t_{n-1}$ is calculated as follows:

$$\varepsilon_{cr}(t_n) = \sum_{i=1}^{n} J'(t_n, t_{i-1})\Delta\sigma(t_{i-1}) \qquad (2)$$

where $\Delta\sigma(t_{i-1})$ is the stress variation during the time change from t_{i-1} to t_i. In Eq. (2), $J'(t_n,t_{i-1})$ is the creep function due to the loading at time t_{i-1} and is calculated from the relationship $J'(t_n,t_{i-1}) = J'(t_n,t_o) - J'(t_{i-1},t_o)$, where $J'(t_n,t_o)$ is the creep function due to the loading at the initial time t_o. Applying the stress increment $\Delta\sigma(t_n)$ at time t_n to Eq. (2), the increment of the creep strain at time $t(t > t_n)$ can be calculated as follows:

$$\Delta\varepsilon_{cr}(t) = \Delta J'\big|_{t=t_n}\left(\sum_{i=1}^{n}\Delta\sigma(t_{i-1}) + \Delta\sigma(t_n)\right) \qquad (3)$$

where $\Delta J'\big|_{t=t_n} = J'(t,t_n) = J'(t,t_o) - J'(t_n,t_o)$.

Based on the horizontal parallel creep curve assumption, the creep strain $\varepsilon_{cr}(t_n)$ at time t_n due to the loadings applied at the ages of $t_o, t_1, t_2, t_3, \ldots, t_{n-1}$ can be calculated as follows:

$$\varepsilon_{cr}(t_n) = \sum_{i=1}^{n} J'(t_n - t_{i-1} + t_o, t_o)\Delta\sigma(t_{i-1}) \qquad (4)$$

Applying the stress increment $\Delta\sigma(t_n)$ at time t_n to Eq. (4), the increment of the creep strain at time t $(t > t_n)$ is calculated as follows:

$$\Delta\varepsilon_{cr}(t) = \sum_{i=1}^{n}\Delta J'_{i-1}(t_n)\Delta\sigma(t_{i-1}) + \Delta J'_n(t_n)\Delta\sigma(t_n) \qquad (5)$$

where $\Delta J'_{i-1}(t_n) = J'(t - t_{i-1} + t_0, t_0) - J'(t_n - t_{i-1} + t_0, t_0)$ represents the increment of the creep function at time t_n due to the loading at time t_{i-1}.

The two limiting cases of Eqs. (3) and (5) are combined to obtain the actual creep curve by introducing an ageing factor $\alpha_n\{= \alpha(t_n)\}$, as follows: