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Modeling geometric imperfections for reticulated shell structures using random field theory

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ABSTRACT

The load-carrying capacity of shell structures and shell-like space frames can be sensitive to initial geometric imperfections. Conventional methods of modeling geometric imperfections have focused on estimating the lower bound of the load-carrying capacity of reticulated shells. This paper proposes a random field model for the initial geometric imperfection of reticulated shell structures. The model accounts for the spatial distribution of the initial geometric imperfections, in which the correlation between imperfections at two different nodes depends on the length and number of the connecting members between the two nodes. The paper also presents the findings of the measurements of the initial geometric imperfections of a real reticulated shell structure. Based on the measurement data, the statistical characteristics of the random field model are determined. The role of initial geometric imperfections on structural ultimate strength is examined using numerical examples.

1. Introduction

Single-layer reticulated shell is widely used in the construction of modern large-span structures, because of its beautiful shape and relatively small weight of structural material per unit area covered. Initial geometric imperfections exist naturally in reticulated shell structures due to construction tolerances, the inaccuracy of measuring equipment, etc. May studies [1–9] have shown that initial geometric imperfections have a significant influence on the strength of shell structures. Even a small amount of initial geometric imperfection may lead to a reduction of over 50% in the load-carrying capacity of the structure [2,10]. To model the initial geometric imperfections rationally in the analysis of large-span reticulated shell structural systems, two important questions have been raised [4]: (1) what is the imperfection mode, and (2) what is the magnitude of the imperfection?

A number of basic methods have been developed to model the geometric imperfections of reticulated shell structures over the past three decades, including the consistent buckling mode method [11], the eigenmode method [12], the Fourier decomposition method [13], and the random imperfection method [14]. In the consistent buckling mode method, the perfect structure is first analyzed by a nonlinear analysis and the nodal displacement increment at the state of incipient collapse is used to represent the mode of the initial imperfection in a subsequent nonlinear analysis. The eigenmode method assumes the imperfect structure has initial displacements in the shape of the elastic critical buckling mode of the structure. In both methods, the initial displacement of the structure is scaled such that the maximum nodal imperfection is equal to a prescribed value, which is $L/300$ in the current Chinese Specification [15], where L represents the largest horizontal dimension of the structure. Fourier decomposition method has been used to interpret the imperfections of cylindrical shells [13,16]. The imperfect surface of a cylindrical shell is described by a series of modal amplitudes and phase angles. Determination of modal amplitudes and phase angles in the Fourier series function requires extensive measurement data, which is often not available in practice.

The random imperfection method recognizes that the initial geometric imperfection is uncertain by its nature. The imperfection at a node is modeled as a random variable. It has been suggested that a normal random variable with a zero mean and a standard deviation equal to half of the construction tolerance can be used to model the nodal geometric imperfection [17]. Samples of imperfect structures can then be generated using Monte Carlo simulation. The minimum value of the strengths of these imperfect structures is taken as the design load-carrying capacity of the structure.

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None of the aforementioned methods is based on rigorous statistical analysis and structural reliability theory. The consistent imperfect ion mode and eigenmode method are deterministic by their nature. The scenarios described by the consistent imperfection mode and the eigenmode methods have very little chance to occur in practice, thus do not provide a realistic way to model the imperfection from a probabilistic point of view. The random imperfection method assumes that the imperfections across the surface of the shell are independent of each other without considering the spatial correlations. This assumption does not reflect the real situation of construction procedure, where the position of a node is likely to affect the position of adjacent nodes as the nodes are connected by steel tubes. Moreover, the current design value for initial geometric imperfection (e.g., $L/300$ in [15]) is empirical and appears to be overly conservative.

The initial geometric imperfection of silo structures has been studied previously, e.g., [18–24]. Measurement data are available for both full-scaled real silo structures [18,21,23] and small-scaled models in laboratory environment [19,20]. However, these measurement data of silos cannot be used for modeling the geometric imperfections of reticulated shells. Silos are continuous cylindrical shells while reticulated shells are discrete structures composed of many steel members. The manufacturing methods and construction process are completely different for these two types of structures. The natures of the initial geometric imperfections for silos and reticulated shells are different.

This paper develops a new random filed approach to model the initial geometric imperfections of reticulated shell structures, considering the randomness and spatial correlation of the imperfection. As part of the present study, the initial geometric imperfections of a real reticulated shell structure were measured on site. The measured data provided valuable information to determine the statistic characteristics of the initial geometric imperfection including spatial correlations. Numerical examples are used to investigate the role of geometric imperfections on structural strength.

The random field model for the initial geometric imperfection is presented in Section 2. Section 3 introduces the measurement results of the geometric imperfections of a real reticulated shell structure. The statistic characteristics of the initial geometric imperfection are discussed in Section 4, followed by numerical examples.

2. Random field model for initial geometric imperfection

2.1. Random field theory

A random field is a family of random variables defined over a spatial structure. Random fields have been used to model random fluctuations in (geometric or material) properties of a structure [25]. The property is a random variable at each location on the structure, and the random properties at two different locations can be mutually correlated. Let $B(\mathbf{u})$ denote a random field. The vector $\mathbf{u}$ is the space coordinates pointing to a location on the structure. In the same way that statistical measures (e.g., mean and variance) are used to represent the statistical characteristics of a random variable, statistical measures are also necessary for the description of a random field. The mean function of $B(\mathbf{u})$ is defined by

$$\mu(\mathbf{u}) = \mathrm{E}[B(\mathbf{u})], \tag{1}$$

in which E[ ] represents the expected value operation. The covariance of the values at two different locations is given by the covariance function:

$$\begin{aligned} C(\mathbf{u}_1, \mathbf{u}_2) &= \mathrm{E}([B(\mathbf{u}_1) - \mu(\mathbf{u}_1)][B(\mathbf{u}_2) - \mu(\mathbf{u}_2)]) \\ &= E[B(\mathbf{u}_1)B(\mathbf{u}_2)] - \mu(\mathbf{u}_1)\mu(\mathbf{u}_2). \end{aligned} \tag{2}$$

The correlation function $\rho(\mathbf{u}_1,\mathbf{u}_2)$ is related to $C(\mathbf{u}_1,\mathbf{u}_2)$ through:

$$\rho(\mathbf{u}_1, \mathbf{u}_2) = \frac{C(\mathbf{u}_1, \mathbf{u}_2)}{\sigma(\mathbf{u}_1)\sigma(\mathbf{u}_2)}, \tag{3}$$

in which σ represents standard deviation, and $C(\mathbf{u}_i,\mathbf{u}_i) = \sigma(\mathbf{u}_i)^2$. A random field is said to be homogeneous (in the weak sense) if its mean is the same at any location, i.e.,

$$\mu(\mathbf{u}) = \text{constant}, \tag{4}$$

and the correlation function depends only on the distance between points, i.e.,

$$\rho(\mathbf{u}_1, \mathbf{u}_2) = \rho(\mathbf{u}_1 - \mathbf{u}_2). \tag{5}$$

Commonly used correlation functions for homogeneous random fields include [26]:

1. Linear correlation function

$$\rho(\mathbf{u}_1, \mathbf{u}_2) = \begin{cases} 1 - \frac{|\mathbf{u}_1 - \mathbf{u}_2|}{a} & |\mathbf{u}_1 - \mathbf{u}_2| \leqslant a, \\ 0, & |\mathbf{u}_1 - \mathbf{u}_2| > a. \end{cases} \tag{6}$$

2. Exponential correlation function

$$\rho(\mathbf{u}_1, \mathbf{u}_2) = \exp\left(-\frac{|\mathbf{u}_1 - \mathbf{u}_2|}{a}\right) \tag{7}$$

3. Gaussian correlation function

$$\rho(\mathbf{u}_1, \mathbf{u}_2) = \exp\left(-\frac{|\mathbf{u}_1 - \mathbf{u}_2|^2}{a^2}\right) \tag{8}$$

in which a is a scaling parameter. Fig. 1 schematically shows the three correlation models. In the linear model, the correlation reduces with distance with a constant rate. The exponential correlation function represents a correlation that decays exponentially with distance. The Gaussian correlation function models a correlation whose decay rate increases with distance. Detailed information about random field theory can be found in a series of books such as [26,27].

2.2. Assumptions for initial geometric imperfection

The initial geometric imperfection considered in this paper is the deviation of the nodes from their nominal locations. The initial geometric imperfection of the members between nodes (i.e., member out-of-straightness) is not considered. The deviation of the position of a node can be either "out-of-surface" or "in-surface". The former represents the nodal deviation in the direction perpendicular to the shell surface, while the in-surface imperfection represents the nodal deviation in the plane of the shell surface. For the

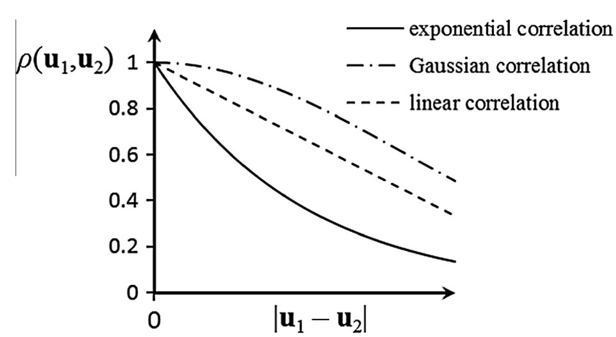

Fig. 1. Typical correlation functions.