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Time-dependent changes in the instantaneous stiffness of reinforced concrete beams

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ABSTRACT

The instantaneous stiffness of a reinforced concrete (RC) beam deteriorates with time and this fact may have important consequences for the long-term in-service behavior of RC structures, particularly those that are subjected to repeated loads, vibrations or dynamic effects. There are two main causes of the time-dependent deterioration of instantaneous stiffness. The first is the formation of new primary cracks, both within and outside of the original cracked region of the beam. The second is the propagation of fine cone-shaped cracks that originate at the steel-concrete interface and are mainly confined within the cover concrete. These cover-controlled cracks facilitate a reduction in bond that is manifested in a decay of tension stiffening within the cracked region of the beam. The formation and propagation of both primary cracks and cover-controlled cracks are driven by the combined effects of shrinkage-induced tensile stress in the concrete and a reduction of the concrete's tensile strength under sustained stress (creep rupture). This paper presents an analytical model for the estimation of the instantaneous stiffness of RC beams with a particular focus on time effects. The model is shown to agree well with recent experimental results.

1. Introduction

In the design of RC beams, satisfaction of the serviceability limit state involves (among other things) the limitation of maximum deflections under normal service loads. The relationship between the maximum deflection and the applied load is governed by the beam's effective flexural rigidity E_cI_{ef}, represented by the secant OB to the moment-curvature envelope OABC for a beam segment, shown in Fig. 1. Point O represents the unloaded and undeformed beam segment before cracking; point A corresponds to the cracking moment M_{cr}, below which the flexural rigidity of the beam segment is equal to that of the uncracked, gross section E_cI_g; and point B is some point within the post-cracking region of the moment-curvature envelope corresponding to a service moment M_{cyc}. Owing to the progressive formation of primary cracks and a decay in the tension stiffening effect with increasing load, the entire moment-curvature envelope OABC for the beam segment is distinctly nonlinear. Simple empirical models for the estimation of effective stiffness are therefore usually favored in design [1,2].

However, when a cracked beam segment is subjected to repeated loads, its flexural behavior departs from the moment-curvature envelope OABC in Fig. 1 and instead follows a new path BD, which for now may be assumed to be linear and to represent a flexural rigidity E_cI_a. Therefore E_cI_a may be thought of as the instantaneous flexural rigidity of the cracked beam segment when subjected to repeated loads, where I_a represents an average second moment of area within the cracked region of a beam. The residual curvature κ_{res}, which remains after the service load is removed, is caused by two main effects: a residual component of reinforcement slip that occurs due to a stiffer unloading bond stress-slip response compared to the initial monotonic loading [3–5]; and the inability of the primary cracks to fully close due to the roughness of the crack faces [5–8].

The instantaneous flexural rigidity E_cI_a and the effective flexural rigidity E_cI_{ef} therefore have rather different meanings and different magnitudes. In situations where serviceability requirements are concerned with the flexural response of already-cracked RC members (e.g. repeated loads, vibrations, dynamic effects), it is the instantaneous flexural rigidity that must be considered. The effective flexural rigidity is of little significance in such cases: its use ignores the residual deformations that develop with the initial loading of the member, leading to underestimation of member stiffness.

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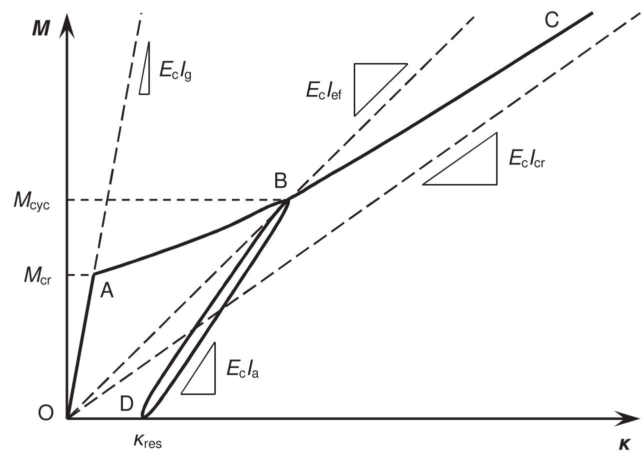

Fig. 1. Moment-curvature relationship for a beam segment subjected to pure bending.

Despite this, only limited guidance is available to engineers for the prediction of the instantaneous response of beams [3,9]. The prediction of a RC beam's instantaneous stiffness is further complicated by its time-dependence, as illustrated by the following example. Fig. 2 shows the relationship between the applied load and the mid-span deflection for one of the RC beams tested in the current experimental program. The beam is initially subjected to a monotonically increasing load P_{cyc} which is then gradually removed and the cycle of loading and unloading is repeated several times. The overall instantaneous stiffness of the beam in the short term (denoted K_0) is defined as the slope of this load-deflection response. Following an extended period of sustained loading (P_{sus}) and exposure to ambient laboratory conditions for several months, the beam is again subjected to several load repetitions and the long-term instantaneous stiffness (denoted K_t) is found. By inspection alone, an obvious time-dependent loss of instantaneous stiffness has occurred (i.e. $K_t < K_0$). A hysteretic effect is observed during the loading and unloading of the beams, though its magnitude is typically small, and the load-deflection response during load repetitions is approximately linear. Hence, the previous assumption of a linear moment-curvature relationship (E_cI_a) for a cracked beam segment subjected to repeated loads is reasonable.

This paper presents an analytical model for the estimation of a RC beam's instantaneous stiffness, with a particular focus on the time-dependent changes in instantaneous stiffness that occur due to the effects of shrinkage, creep, and sustained loading. The model presented in this paper will assist engineers to satisfy short-term and long-term serviceability requirements for RC beams in cases where conventional effective stiffness models do not apply.

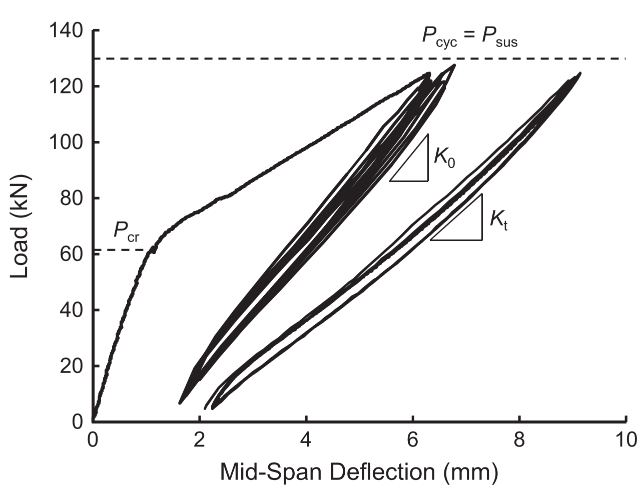

Fig. 2. Load-deflection relationship for S2-B6 showing instantaneous stiffness in the short term (K_0) and long term (K_t).

2. Experiments

The experimental program consisted of two series of tests (designated S1 and S2). The aim of the experiments was to determine the instantaneous stiffness of cracked RC beams in the short term (K_0), as well as after an extended period of sustained loading and exposure to ambient laboratory conditions (K_t). The first test series consisted of six beams labelled S1-B1 to S1-B6, and is described in detail in [10]. The second (more recent) series was conducted by the same authors and included eight beams labelled S2-B1 to S2-B8. This paper considers the results of thirteen of these fourteen beams. Beam S1-B4 is excluded here since its testing was restricted to the short term only.

All thirteen beams considered in this paper were 3.5 m in length and spanned 3.3 m between simple (pin and roller) supports. The beams were subjected to four-point bending with equal point loads applied at the third points (Fig. 3a). Each beam had a rectangular cross-section with dimensions of 400 × 300 mm. The longitudinal reinforcement consisted of three ribbed steel reinforcing bars having nominal diameters of either 16 mm or 20 mm and a characteristic yield strength of f_{sy} = 500 MPa. These reinforcement arrangements correspond to reinforcement ratios ρ ($\rho = A_s/bd$) within the range of 0.56–0.88%. The clear concrete cover to the longitudinal reinforcement was 35 mm for all beams. Within the two shear spans, two-legged closed stirrups of 8 mm diameter were spaced at 200 mm intervals; however, no stirrups were placed in the constant bending moment region so as not to unduly influence the spacing of primary cracks. The reinforcement layout for a typical beam is shown in Fig. 3b and details are provided in Table 1. The longitudinal reinforcement for beam S2-B4 consisted of two 16 mm bars plus one 20 mm bar; for simplicity in calculations, it is assumed that the beam contains three bars of an equivalent diameter of 17.4 mm, which provides the same cross-sectional area of reinforcement.

All beams were fabricated with normal-strength ready-mix concrete using plywood forms which were stripped 7 days after casting. For series S1, beams S1-B1 to S1-B3 were continuously moist cured for a period of about one month until first loading in order to prevent drying shrinkage strains from developing prior to short-term testing. The other two beams from this series (S1-B5 and S1-B6) were moist cured for a period of 7 days and were

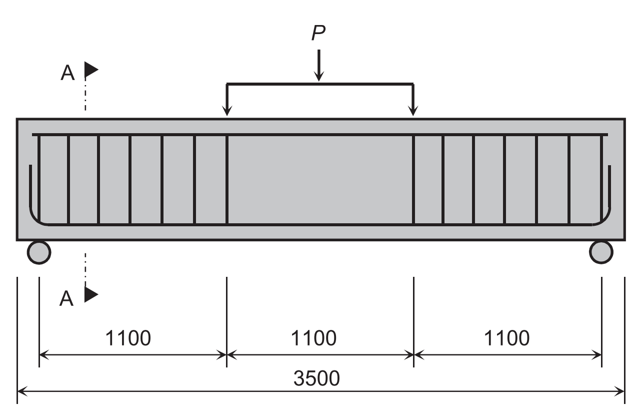

Fig. 3a. Elevation of typical beam.