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Composite beam theory for pretensioned concrete structures with solutions to transfer length and immediate prestress losses

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ABSTRACT

A composite beam theory is developed to study the pretensioned concrete structures. This theoretical development defines the longitudinal interaction that occurs between the prestressing tendon and concrete under normal service condition. The transfer length and prestress loss due to slip, elastic shortening as well as the prestress gain due to external loads are solved with closed form solutions. Validation is provided and comparisons are made between the present and conventional approaches. It is found that there is excellent agreement between the predicted and tested concrete strain. The transfer length results calculated by design provisions are close to the predicted upper bound but test results can be anywhere between the predicted lower and upper bounds. Current immediate prestress losses formulas may result in overestimation and the degree of overestimation is dependent on prestressing tendon eccentricity. Overall, it is demonstrated that the approach presented in this study improves the accuracy and facilitates a better understanding of prestressed concrete mechanics while maintaining concise closed form solutions.

1. Introduction and literature review

Although the idea of prestressing has been around for hundreds of years [1], modern prestressed concrete structures are considered to be officially introduced by Freyssinet to the public in 1936 [2,3]. The first prestressed concrete bridge in the United States, Walnut Lane Memorial Bridge designed by Gustave Magnel [4], was completed in 1950 [5–7]. The construction of Walnut Lane Memorial Bridge drew attention to prestressed concrete structures and, to some extent, initiated the modern prestressed concrete industry in the United States. Major prestressed concrete design codes and specifications in the United States started to shape out in 1950s and 1960s [8–10]. After decades of development, nowadays prestressed concrete structure, as one of the most important topics in the structural engineering, still poses some challenges in defining its behaviors. Fundamental issues such as transfer length of the prestressing force and prestress losses in the pretensioned concrete structures are still under development. This study adopts a composite beam theory for pretensioned concrete analysis, and transfer length and immediate prestress losses are solved with closed form solutions herein.

1.1. Composite beam theory

The composite beam theory was originally developed for composite structures such as insulated sandwich structures, steel concrete composite T girders, and nailed timber structures [11,12], but as demonstrated herein, by incorporating the prestressing effect and other unique features of prestressing tendons it can also be applied to pretensioned concrete. The composite beam theory first considers the prestressing tendon and concrete independently so that their individual properties can be implemented. Then the force equilibrium between the prestressing tendon and concrete is established through their interface properties, for example, load-slip relationship. Granholm [13] published the earliest composite beam theory for the nailed timber structures in 1949 and two years later Newmark et al. [14] published his work on composite T girders. Although Granholm and Newmark independently published their works on the composite beam theory and their theories were also derived from different angles, Goodman [15] proved that their works are comparable. Holmberg and Plem [16] adopted and improved Granholm's theory and applied it to concrete sandwich structures. Girhammar and Gopu [17] developed a second order model and provided closed form solutions. Adekola [18] presented a numerical model to account for transverse interaction. Foraboschi [19,20] developed analytical models for two-layer composite beams and three-layer sandwich composite plates.

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Nomenclature

φ_{2c}	concrete beam displacement due to axial force
φ_{2s}	tendon displacement due to axial force
E_c	concrete elastic modulus
E_s	prestressing tendon elastic modulus
q	shear force per unit length within the concrete-tendon interface
N_c	resultant axial force in the concrete beam
A_c	cross section area of the concrete beam
φ_2	total slip between concrete and tendon due to axial force
N_s	resultant axial force in the tendon
N	resultant axial force in both the concrete beam and tendon
K	shear bond stiffness of the concrete-tendon interface
φ	$\varphi_1 + \varphi_2 + \varphi_3$, total slip between concrete beam and tendon
φ_1	ey', slip due to bending
φ_3	$\int \varepsilon_{is}\, dx$, slip due to prestress
e	the distance from the concrete beam centroid to the tendon centroid
y	deflection
η	$A_cE_cA_sE_s/(A_cE_c + A_sE_s)$
χ_B^2	K/η
M_{ex}	external applied bending moment
M_c	internal bending moment of the concrete beam
I_c	concrete beam moment of inertia
ε_{is}	strain resulted from prestress, before transfer, in the prestressing tendon
D_B	$E_cI_c + e^2\eta$
α_B^2	$e^2\eta/D_B$
q_w	uniformly distributed pressure of unit length
β_B^2	$1 - \alpha_B^2$
φ_r	total slip between concrete beam and tendon of the reinforced concrete beam
N_r	resultant axial force of the reinforced concrete beam
M_r	internal bending moment of the reinforced concrete beam
φ_p	total slip between concrete beam and tendon of the prestressed concrete beam
N_p	resultant axial force of the prestressed concrete beam
M_p	internal bending moment of the prestressed concrete beam
ε_{cds}	eM_{ex}/D_B
f_{cds}	$eM_{ex}E_c/D_B$
$\Delta N_{P\text{-}ES}$	change in axial force due to elastic shortening without slip
$P_{effe\text{-}ES}$	effective prestress after elastic shortening loss in percentage
$P_{loss\text{-}ES}$	prestress loss due to elastic shortening in percentage
$\Delta N_{P\text{-}ES\text{-}sp}$	change in axial force due to elastic shortening considering slip
$\Delta N_{P\text{-}sp}$	change in axial force due to slip
$P_{loss\text{-}sp}$	prestress loss due to slip in percentage
γ	the tolerance of axial force at mid-span
L_t	transfer length
λ_t	$(1-\gamma)\cosh(\chi_B l/(2\beta_B))$
φ_{pe}	$\varphi_p(l/2)$, slip at the end due to prestressing force
μ	$\chi_B l/(2\beta_B)$
f_{se}	effective prestressing force in psi
d_b	strand nominal diameter
K_{es}	1.0 for pretensioned components
f_{cir}	net compressive stress in concrete at the center of gravity of prestressing force immediately after the prestress has been applied to the concrete
K_{cir}	0.9 for pretensioned components
N_i	initial prestressing force
M_c	bending moment due to dead weight of the prestressed component and any other permanent loads in place at the time of prestressing

Focacci et al. [21] further developed Foraboschi's [19] model by implementing numerical schemes, and compared and discussed the difference between the two approaches. Bai [12] decoupled the longitudinal and transverse interactions and derived closed form solutions for both. Ranzi [22,23] and Zona and Ranzi [24] developed designated composite elements for finite element analysis. Xu and Wu [25] derived closed form solutions based on plane stress assumption.

1.2. Transfer length

Prestressing force transfer is a complicated but interesting topic. Transfer length is the distance required to transfer the effective prestressing force from prestressing tendons to the concrete; in other words, it is the distance that concrete axial force increases from zero to a constant value after prestressing force release, as illustrated in Fig. 1. In pretensioned concrete structures, the prestressing forces are primarily transferred to the concrete within the relatively short transfer zones; away from the transfer zones the prestressing tendons have no interaction with the concrete as the concrete/tendon interface shear force becomes zero (also illustrated in Fig. 1). The transfer zones essentially serve as anchorages, and the prestressing force out of transfer zones is transferred by the "anchorage" concrete in the transfer zones instead of prestressing tendons. Load behavior wise, if the transfer length is short as in beams with good bonding, the concrete near the end of the prestressing tendons will be susceptible to cracking and procedures such as tendon debonding or draping may be needed. However, if the transfer length is too long as in the poorly bonded beams, the shear capacity may be compromised. Therefore, a reliable prediction of the bonding condition and transfer length is essential to the success of pretensioned concrete structures.

At the early stage of prestressed concrete development, transfer length was studied to answer the question of whether a mechanical locking device should be used to ensure that prestressing forces can be fully transmitted to the concrete in pretensioned concrete construction [26,27]. Janney [28] described the complicated prestress transfer bond behaviors in a series of tests and studied the influences of strand diameter, surface condition and concrete strength. As the material properties and production process of both concrete and prestressing tendon evolved in the prestressing concrete industry, transfer length has been continuously investigated by a large number of studies. Russell [29–31], Barnes [32,33], Oh and Kim [35], Kim [34], Zia and Mostafa [36] and Marti-Vargas [37,38] all conducted tests on transfer length and discussed a number of factors that can influence transfer length. Peterman [39,40], Floyd et al. [41] and Boehm et al. [42] tested the transfer length of self-consolidating concrete structures and light weight concrete. Kahn examined the transfer length on high performance concrete girders [67]. Osborn et al. [43] evaluated the bonding condition