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Thermal convection in a toroidal duct of a liquid metal blanket. Part II. Effect of axial mean flow

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HIGHLIGHTS

- 2D convection flow develops with internal heating and strong axial magnetic field.
- The flow is strongly modified by the buoyancy force associated with growing T_m.
- Thermal convection is suppressed at high Gr.
- High temperature difference between top and bottom walls is expected at high Gr.

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ABSTRACT

The work continues the exploration of the effect of thermal convection on flows in toroidal ducts of a liquid metal blanket. This time we consider the effect of the mean flow along the duct and of the associated heat transfer diverting the heat deposited by captured neutrons. Numerical simulations are conducted for a model system with two-dimensional (streamwise-uniform) fully developed flow, purely toroidal magnetic field, and perfectly electrically and thermally insulating walls. Realistically high Grashof (up to 10^{11}) and Reynolds (up to 10^6) numbers are used. It is found that the flow develops thermal convection in the transverse plane at moderate Grashof numbers. At large Grashof numbers, the flow is dominated by the top-bottom asymmetry of the streamwise velocity and stable stratification of temperature, which are caused by the buoyancy force due to the mean temperature growing along the duct. This leads to suppression of thermal convection, weak mixing, and substantial gradients of wall temperature. Further analysis based on more realistic models is suggested.

1. Introduction

As discussed in detail in our earlier work [1] and in the accompanying paper [2], the focus of our study is on the effect of thermal convection on the flow of a liquid metal in a horizontal duct with strong axial magnetic field. We revisit, on a conceptual level, the old idea of a liquid metal blanket with toroidal ducts (see, e.g. [3,4] and the chapter by L. Bühler in [5]). The earlier analysis of such a blanket was, in our view, decisively incomplete, since the effect of the thermal convection on the flow was ignored.

Using the idealized model of a segment of a duct with periodic inlet-exit, we show, in [1], that the strong toroidal magnetic field typical for the reactor conditions makes the flow purely or nearly two-dimensional. At the same time, turbulence is generated by the thermal convection caused by the non-uniform heating of the liquid metal by the absorbed neutrons. This provides a mechanism for efficient transfer of heat toward the walls of the duct and removes the effect of high-amplitude low-frequency 'anomalous' fluctuations of temperature detected recently in poloidal ducts [6–9].

The analysis is continued in [2], where we consider the effect of the additional weaker vertical magnetic field. We find that such field suppresses turbulence but not the convection-caused circulation, which becomes dominated by large oscillating or steady vortices. The flow retains its abilities to transport heat toward walls and avoid anomalous fluctuations.

The idealized model used in [1,2] includes the assumption that the energy deposited into the liquid metal by internal heating is removed via cooling of the walls, which are maintained at a constant temperature. The mean flow along the duct is assumed zero. In this paper, we analyze a configuration, which, while still strongly idealized, is more realistic and closer to the concepts of self-cooled and dual-coolant blankets. We assume that the walls are thermally

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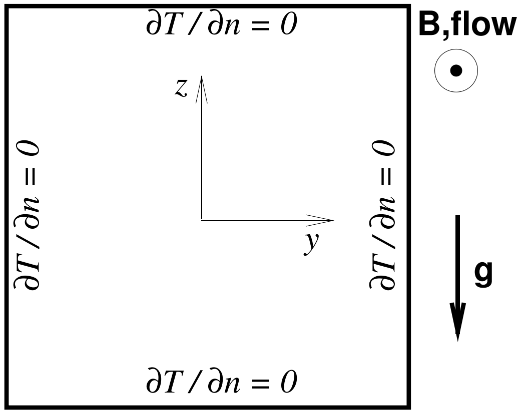

Fig. 1. Geometry of the flow and coordinate system. T is the temperature field.

perfectly insulating and that the heat is removed by the mean flow along the duct.

2. Physical model and numerical method

We consider a liquid metal (an incompressible, viscous, electrically conducting fluid) flowing inside a horizontal duct with imposed magnetic field $\boldsymbol{B}$ (see Fig. 1). The magnetic field is assumed uniform, static, and perfectly aligned with the axis of the duct. In that approach, we follow [1], neglecting the effect of the poloidal magnetic field at the stage of the study. The duct walls are thermally and electrically insulating. The strong internal heating due to absorption of neutrons is modeled as non-uniform internal heating of volumetric rate $Q_0q(y)$, where Q_0 is a constant and y is the distance from the wall nearest to the reaction zone (see Fig. 1c of [2] for the profile of $q(y)$).

The duct is assumed to be long, and a fully-developed flow is considered. As an approximation, we assume that, due to the strong suppression by the magnetic field, the flow can be modeled as two-dimensional (uniform in the axial direction). This means that there is no active (non-potential) Lorentz force in our model.

The validity of the two-dimensional (2D) approximation is based on the findings of [1,2] that in the range of parameters typical for a fusion reactor blanket (the Hartmann number of the order of 10^4 and the Grashof number up to 10^{11}), the flow in a model of a long toroidal duct, in which the end effects are neglected, is either purely 2D or with weak three-dimensional (3D) perturbations that do not affect the flow properties in a major way. Our 3D computations conducted in a domain of length 4π for the physical model considered in this paper here produced similar results [17].

We use the typical scales similar to those in [2]: the duct's half-width d and $\Delta T = Q_0 d^2\kappa^{-1}$ are used to scale the length and temperature. We use the mean flow axial velocity U_m as the velocity scale. The time scale is d/U_m. The non-dimensional governing equations are:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p - \nabla\hat{p} - \nabla\tilde{p} + \frac{1}{Re}\nabla^2\boldsymbol{u} + \boldsymbol{F}_b, \tag{1}$$

$$\nabla\cdot\boldsymbol{u} = 0, \tag{2}$$

$$\frac{\partial\theta}{\partial t} + \boldsymbol{u}\cdot\nabla\theta = \frac{1}{Pe}(\nabla^2\theta + q) - u_x\frac{dT_m}{dx}. \tag{3}$$

The non-dimensional internal heating rate is modeled as $q = \exp[-(y+1)]$ [10]. The non-dimensional parameters are the Grashof number $Gr = g\beta Q_0 d^5/\kappa\nu^2$, the Reynolds number $Re = Ud/\nu$ and the Peclet number $Pe = RePr$, where $Pr = \nu/\chi$ is the Prandtl number. Here, β, κ, ν, χ are, respectively, the thermal expansion coefficient, thermal conductivity, kinematic viscosity and temperature diffusivity of the fluid. As there is no active Lorentz force acting on the flow, Hartmann number and electric currents do not appear in our governing equations and boundary conditions. The temperature field is the sum

$$T(\boldsymbol{x}, t) = T_m(x) + \theta(y, z, t) \tag{4}$$

of fluctuations θ and the mean-mixed temperature

$$T_m(x) = \frac{1}{A}\int_A u_x T dA, \tag{5}$$

where A is the cross-section area of the duct. Applying the energy balance between the volumetric internal heating and the heat transfer by axial flow, we find

$$\frac{dT_m}{dx} = \frac{Q}{APe} = \frac{Q}{APrRe} = const > 0, \tag{6}$$

where $Q = \int_A q dA$ is the non-dimensional internal heating rate per unit length of the duct. It can be easily shown that the mean temperature obtained by averaging of T over the duct's cross-section grows with the same gradient.

The buoyancy term in (1) is computed as:

$$\boldsymbol{F}_b = \frac{Gr}{Re^2}\theta\boldsymbol{e}_z. \tag{7}$$

According to the definition of temperature field in (4), the total buoyancy force is a sum of $\boldsymbol{F}_b$ and $\tilde{\boldsymbol{F}}_b$, where $\tilde{\boldsymbol{F}}_b$ is associated with the mean-mixed temperature T_m and is discussed later.

The boundary conditions at the walls are those of thermal insulation

$$\frac{\partial\theta}{\partial n} = 0 \quad \text{at} \quad y = \pm 1, \quad z = \pm 1, \tag{8}$$

and no-slip

$$\boldsymbol{u} = 0 \quad \text{at} \quad y = \pm 1, \quad z = \pm 1. \tag{9}$$

In three-dimensional simulations performed to validate the 2D approximation, periodic boundary conditions at the inlet and exit of the flow domain are used.

The total pressure field P is a sum

$$P = \hat{p}(x) + \tilde{p}(x, z) + p(y, z, t). \tag{10}$$

Here, $\hat{p}$ is a linear function of x. The corresponding spatially uniform streamwise gradient $d\hat{p}/dx$ is used as a flow-driving mechanism in our model. The value of $d\hat{p}/dx$ is adjusted at every time step to maintain constant mean velocity equal to one. As discussed in [11–14], the rest of the decomposition becomes necessary in numerical models of mixed convection in non-vertical channels with periodic inlet-exit conditions or, as in our case, with the axially uniform flows. The field p is that of the perturbations of pressure. It has zero mean and does not vary in the axial direction. The need for

$$\tilde{p}(x, z) = \frac{dT_m}{dx}\frac{Gr}{Re^2}xz = \frac{QGr}{ARe^3Pr}xz \tag{11}$$

arises due to the buoyancy force

$$\tilde{\boldsymbol{F}}_b = \frac{Gr}{Re^2}T_m(x)\boldsymbol{e}_z \tag{12}$$

caused by the mean-mixed temperature T_m. The force increases linearly with x and has constant non-zero curl. The force appears in the governing equations describing three-dimensional flows evolving along the duct (see, e.g. [13]). Considering the problem for axially periodic or uniform perturbations of the flow fields θ, $\boldsymbol{u}$, we can fully take this force into account by following the approach of [11,12] (see also our recent work [14]). The component of the pressure field $\tilde{p}$ is separated from the rest of the field, such that its vertical gradient $\partial\tilde{p}(x, z)/\partial z = GrRe^{-2}T_m(x)$ accounts for the force $\tilde{\boldsymbol{F}}_b$. In