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Optimal control problem in bond graph formalism

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ABSTRACT

This paper presents a new way to derive an optimal control system for a specific optimisation problem, based on bond graph formalism. The procedure proposed concerns the optimal control of linear time invariant MIMO systems and can deal with both cases of the integral performance index, these correspond to dissipative energy minimization and output error minimization. An augmented bond graph model is obtained starting from the bond graph model of the system associated with the optimal control problem. This augmented bond graph, consisting of the original model representation coupled to an *optimizing bond graph*, supplies, by its bicausal exploitation, the set of differential-algebraic equations that analytically give the solution to the optimal control problem without the need to develop the analytical steps of Pontryagin's method. The proof uses the Pontryagin Maximum Principle applied to the port-Hamiltonian formulation of the system.

1. Introduction

Bond graph language proves to be a very efficient tool for modelling, analysing and designing mechatronic systems from an energy and dynamic point of view [4,11]. The main idea presented in this paper is to introduce an optimal control problem into bond graph formalism. The perspective is to couple this formulation with a sizing methodology of mechatronic systems using bond graph language and the state space inverse model approach. This methodology of sizing, based on dynamic and energy criteria using bond graph language, was developed at the Laboratoire d'Automatique Industrielle[1] [5–8]. The objective is to transpose the optimization problem into bond graph formalism so that its exploitation will solve this problem. The procedure for building the bond graph representation of the given optimal control problem is presented; this enables the set of differential-algebraic equations to be derived that give the solution to the optimal control problem. In fact, the equations obtained are derived graphically by assigning the bicausality to this augmented bond graph representation, avoiding the analysis usually involved by applying the Pontryagin Maximum Principle [12].

This paper lays out all our research work published until now [3,13,15,16], moreover this research presents the propositions within a more general framework of linear time invariant MIMO systems. This research was started by some of the authors of this paper and the first results were given at the 7th ICBGM conference in New Orleans [13]. This paper consisted of building an augmented bond graph from the bond graph model of a system, where the assignment of the bicausality gives the solution to the optimal control problem. The method has been developed using the example of a DC motor and the optimization objective taken was the minimization of dissipative energy. The optimisation objective is expressed as the integral of a quadratic form of the state space vector and the control input to be determined. The optimal control problem was

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[1] Since January 1, 2007, the LAI, the CEGELY and a team of environmental genomic microbial have been regrouped to be called 'Ampère'.

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formulated analytically and, in parallel, by means of bond graph language. This very simple example has been chosen to understand better the whole bond graph generation mechanism. The steps of bond graph formulation specify a systematic procedure. This procedure aims at automatically generating the corresponding bond graph representation and a proof of its effectiveness. It has been presented in [15]. The class of system that the procedure can deal with was formerly restricted to linear time invariant SISO systems before it was extended to linear time invariant MIMO systems. Recently, a new investigation of the bond graph construction of an optimal control problem for another performance index has been carried out. This studies the output trajectory tracking [16]. In this case, the performance index may be expressed as a quadratic form of error to be minimized between a specified output and the actual output. The control variable is also taken into account by using a weighting factor. Here, the two procedures are grouped into only one procedure. This enables the reader to apply his optimal control problem without returning to the previous references and it takes into account the various types of the input and output (effort or flow).

The key idea of the proof of this procedure is to apply the Pontryagin Maximum Principle to a generic port-Hamiltonian system. Port-Hamiltonian system is an analytical expression of the dynamic equations governing a model that mathematically clearly reflects the energy topology of the system model [14,17]. Boundary conditions are supposed fixed, in particular for both final time and final state and, finally, no constraint exists on either inputs or states. This voluntary restricted hypothetical framework has enabled the first step in the coupling of optimisation and bond graph to be clearly investigated and offers encouraging perspectives for future work. A simple numerical method for solving the problem of finding the initial costate conditions from the initial set of boundary conditions has been implemented and is given in [3]. This point will not be detailed in this paper and the reader will be able to consult the reference above for more details.

2. Organization of the paper

This paper is organized as follows. Section 3 synthesizes the procedure of the bond graph construction of an optimal control problem in the form of a proposition where the integral performance index may contain dissipative energy and/or an output error to minimize. This gives the construction steps of this augmented bond graph representation and it also summarizes the conditions of how the procedure can be applied. Its demonstration justifies the former procedure and proves its effectiveness. This is given in Section 4. The developments concerning the optimal control problem are based on the Pontryagin Maximum Principle and the proof of the procedure's effectiveness uses the port-Hamiltonian concept.

The proposed graphical procedure is tested on an example of three masses in series shown in Section 5. This graphical procedure shows that we can obtain the same result as the one that we would obtain by using classical analytic developments. Section 6 concludes the paper with a summary of the results and suggests future directions of research.

Additionally the two appendices give, respectively, bases on partial dualization in the bond graph used for proof of the effectiveness of the procedure (Appendix A) and on the bicausality concept for the bond graph exploitation that provides the optimal control system (Appendix B).

3. Procedure for the construction of a bond graph optimal control problem

This section proposes a systematic procedure for generating the bond graph of the optimal control problem within a general framework of linear time invariant MIMO systems when the optimisation objectives are minimizing the dissipation and the tracking of a reference trajectory. The control vectors to be determined can be effort and/or flow variables, the same is true for the nature of the output vectors for tracking/specifying. Thus, this optimal control bond graph representation would be exploitable for directly determining the optimal control solution without developing the analytical steps of Pontryagin's method. However, due to the relative novelty of the approach presented, it is essential to recall its conditions of application:

Let:

- a linear time invariant model of a MIMO system and its bond graph representation,
- the input controls to determine with respect to the integral performance index to minimize. This integral is the terms of half of a control-based quadratic form, a dissipative energy, and/or a quadratic error between specified outputs and the actual outputs,
- fixed-boundary conditions for the time and state space and
- no constraint on inputs or on state.

So, knowing a trajectory defined by $\{y_r(t)\}_{t\in[t_0,t_f]}$, where t_0 and t_f indicate the horizon of the fixed state, the problem is to determine the control u for the given initial states x_0. This control minimizes some dissipative energy (power P_{diss}) while keeping the output error $y(t) - y_r(t)$ bounded. This problem can be formalized as a problem of quadratic error minimization on the time boundary $[t_0, t_f]$:

$$V = \int_{t_0}^{t_f} \frac{1}{2}\left[u^T \cdot R_u^{-1} \cdot u + P_{diss} + (y - y_r)^T \cdot Q \cdot (y - y_r)\right] dt \quad (1)$$