Fusion Engineering and Design 117 (2017) 74–78

Contents lists available at ScienceDirect

Fusion Engineering and Design

journal homepage: www.elsevier.com/locate/fusengdes

Determination of elastic modulus for hollow spherical shells via resonant ultrasound spectroscopy

Xiaojun Ma [a,b], Xing Tang [b], Zongwei Wang [b], Qian Chen [c], Menglu Qian [c], Jie Meng [b], Yongjian Tang [a,b], Hao Shen [a], Dangzhong Gao [b,*]

[a] Institute of Modern Physics, Fudan University, Shanghai 200433, China
[b] Research Center of Laser Fusion, CAEP, Mianyang 621900, China
[c] Institute of Acoustic, Tongji University, Shanghai 200092, China

HIGHLIGHTS

- The axisymmetric frequency equation of an isotropic hollow two-layer sphere is deduced by three dimension elasticity theory and global matrix method.
- The simulated results demonstrate that the natural frequencies of a hollow sphere are more strongly dependent on Young's modulus than Poisson's ratio.
- The Young's moduli of polymer capsules with an sub-millimeter inner radius are measured accurately with an uncertainty of ~10%.

ARTICLE INFO

Article history:
Received 18 July 2016
Received in revised form 10 January 2017
Accepted 14 February 2017

Keywords:
Inertial confinement fusion
Resonant ultrasound spectroscopy
Elastic modulus
Capsule

ABSTRACT

The elastic property of a capsule is one of the essential parameters both in engineering applications and scientific understanding of material nature in inertial confinement fusion (ICF) experiments. The axisymmetric frequency equation of an isotropic hollow two-layer sphere is deduced by three dimension elasticity theory and global matrix method, and a combined resonant ultrasound spectroscopy(RUS), which consists of a piezoelectric-based resonant ultrasound spectroscopy(PZT-RUS) and a laser-based resonant ultrasound spectroscopy(LRUS), is developed for determining the elastic modulus of capsule. To understand the behavior of natural frequencies varying with elastic properties, the dependence of natural frequencies on Young's modulus and Poisson's ratio are calculated numerically. Some representative polymer capsules are measured using PZT-RUS and LRUS. Based on the theoretical and experimental results, the Young's moduli of these capsules are measured accurately with an uncertainty of ~10%.

1. Introduction

In inertial confinement fusion (ICF) experiments, a hollow spherical shell made of polymer (CH), which is generally called a capsule, is one of the alternative ignition design capsules [1]. To reach the conditions needed for ignition, many specifications of the capsule, such as geometrical structures, outer and inner surface roughness, dopant concentration and deuterium-tritium (D-T) fuel content must meet a rigorous designed standard [2]. In these parameters, the elastic modulus of material is a key parameter in both engineering application and science understanding for the physical nature of material. From the viewpoint of engineering applications, the elastic modulus directly determines the deformation behavior of capsule under high pressure where the DT fuel is filled in the capsule and low temperature conditions where the DT gases will be cooled to a solid DT ice layer. Moreover, the elastic modulus is closely related to the interaction between atoms, and the knowledge of elastic properties of material will provide a way to understand the physical nature of material [3]. Consequently, it is of particular importance to measure accurately the elastic modulus of ICF capsule in a nondestructive manner.

Since resonant ultrasound spectroscopy (RUS) is developed by Migliori and his co-workers at Los Alamos National Laboratory [4], it has been widely applied to characterize the samples with known geometries [5–7]. The internal gas density and pressure in metal capsules was also determined accurately by this technique [8,9]. As for an ICF capsule, it is a typical spherical shell with high symmetry, where an accuracy of a few parts in 10^4 is expected. Therefore, it is easy to model with an extra accuracy and the natural frequencies of capsule can be accurately calculated. Using RUS technique, the

* Corresponding author.
E-mail address: dgaocn@163.com (D. Gao).

http://dx.doi.org/10.1016/j.fusengdes.2017.02.050

elastic modulus of capsule may be determined accurately. However, little attention has been given to evaluate the elastic properties of the hollow layered spherical sphere.

In this paper, the characteristic equation of an isotropic hollow two-layer sphere was firstly derived under the axisymmetric condition by three-dimensional (3D) elasticity theory and global matrix method. Then, the dependence of natural frequencies on Young's modulus and Poisson's ratio are calculated numerically, and two representative capsules were measured with a combined RUS apparatus, which consists of piezoelectric-based resonant ultrasound spectroscopy (PZT-RUS), and laser-based resonant ultrasound spectroscopy (LRUS). Lastly, the elastic moduli of capsules were determined from the measured resonant frequencies as an inverse problem of the frequency equation.

2. Theoretical analysis

According to Helmholtz theorem, the vector field can be expressed in terms of the gradient of a scalar potential and the curl of a vector potential. In axisymmetric condition, the displacement vector u in spherical coordinates (r, θ, φ) can be expressed by a scalar potential ϕ related to the longitudinal waves and a vector potential ψ associated with the transverse waves.

$$u(r,\varphi,\theta) = u(r,\theta) = \nabla\phi + \nabla\times\psi \tag{1}$$

$$\nabla^2\phi = \frac{1}{{c_1}^2}\frac{\partial^2\phi}{\partial t^2} \tag{2}$$

$$\nabla^2\psi - \frac{\psi}{r^2\sin^2\theta} = \frac{1}{{c_2}^2}\frac{\partial^2\psi}{\partial t^2} \tag{3}$$

with ${c_1}^2 = {(\lambda+2\mu)}/{\rho}$, ${c_2}^2 = {\mu}/{\rho}$. $c_1$and c_2 are the velocities of longitudinal and transverse waves, ρ is the mass density, λ and μ are the Lame's constants. ∇ and ∇^2 are the usual del operator and Laplace operator, respectively.

Using separation variable technique, the solutions to Eqs. (2) and (3) can be written as

$$\begin{aligned} u_r &= \sum_{m=0}^{\infty} k_1[A_m {j_m}'(k_1r) + B_m {n_m}'(k_1r)] - \frac{1}{r}m(m+1)[C_m j_m(k_2r) + D_m n_m(k_2r)]P_m(\cos\theta) \\ u_\theta &= \sum_{m=0}^{\infty}\{\frac{1}{r}[A_m j_m(k_1r) + B_m n_m(k_1r) - C_m j_m(k_2r) - D_m n_m(k_2r)] - k_2[C_m {j'}_m(k_2r) + D_m {n'}_m(k_2r)]\}\frac{dP_m(\cos\theta)}{d\theta} \end{aligned} \tag{4}$$

where k_1 and k_2 are the wave vector of longitudinal and transverse waves, respectively. $j_m(kr)$and $n_m(kr)$ are the spherical Bessel functions of the first and second kinds, respectively. m is the order of Bessel function. $P_m(\cos\theta)$ is the Legendre polynomial. A_m, B_m, C_m and D_m, are arbitrary constants.

Considering the natural modes are independent, according to the stress-displacement relation, the stress field can be given by

$$\sigma_{rr} = x_{11}A_m + x_{12}B_m + x_{13}C_m + x_{14}D_m \tag{5}$$

$$\sigma_{r\theta} = x_{21}A_m + x_{22}B_m + x_{23}C_m + x_{24}D_m \tag{6}$$

where

$$x_{11} = [(\lambda+2\mu){k_1}^2{j_m}''(k_1r) + \frac{2\lambda}{r}k_1{j_m}'(k_1r) - \lambda\frac{m(m+1)}{r^2}j_m(k_1r)]P_m(\cos\theta)$$

$$x_{12} = [(\lambda+2\mu){k_1}^2{n_m}''(k_1r) + \frac{2\lambda}{r}k_1{n_m}'(k_1r) - \lambda\frac{m(m+1)}{r^2}n_m(k_1r)]P_m(\cos\theta)$$

$$x_{13} = 2\mu[\frac{1}{r^2}m(m+1)j_m(k_2r) - \frac{k_2}{r}m(m+1){j_m}'(k_2r)]P_m(\cos\theta)$$

$$x_{14} = 2\mu[\frac{1}{r^2}m(m+1)n_m(k_2r) - \frac{k_2}{r}m(m+1){n_m}'(k_2r)]P_m(\cos\theta)$$

$$x_{21} = 2\mu[-\frac{1}{r^2}j_m(k_1r) + \frac{k_1}{r}{j_m}'(k_1r)]\frac{dP_m(\cos\theta)}{d\theta}$$

$$x_{22} = 2\mu[-\frac{1}{r^2}n_m(k_1r) + \frac{k_1}{r}{n_m}'(k_1r)]\frac{dP_m(\cos\theta)}{d\theta}$$

$$x_{23} = \mu[-\frac{1}{r^2}(m^2+m-2)j_m(k_2r) - {k_2}^2{j''}_m(k_2r)]\frac{dP_m(\cos\theta)}{d\theta}$$

$$x_{24} = \mu[-\frac{1}{r^2}(m^2+m-2)n_m(k_2r) - {k_2}^2{n''}_m(k_2r)]\frac{dP_m(\cos\theta)}{d\theta}$$

For a hollow single layer sphere, the free boundary conditions are taken as

$$\sigma_{rr}(r) = \sigma_{r\theta}(r) = \sigma_{r\phi}(r) = 0\,(r = a, b) \tag{7}$$

Where a and b are the inner and outer radius of the sphere, respectively.

Thus, the characteristic equation can be expressed as

$$|x_{ij}|_{4\times4} = 0 \tag{8}$$

Where x_{ij} can be obtained by instead of a and b in the expressions just given in Eqs. (5) and (6).

As for a hollow two-layer sphere, according to the free boundary conditions at inner and outer surface and continuing boundary conditions at interface, the characteristic equation can be rewritten as

$$|x_{ij}|_{8\times8} = 0 \tag{9}$$

Where x_{ij} can be obtained by instead of radius values at inner and outer surface and interface in the expressions just given in Eqs. (4)–(6).

Based on the frequency Eqs. (8) and (9), the natural frequencies of a hollow one-layer or two-layer capsule can be calculated exactly. Utilizing the measured natural frequencies, the elastic modulus of capsule can be determined as an inverse problem of Eqs. (8) and (9).

As for an elastic solid sphere with a free boundary, the frequency equation reduces to the expression [10]:

$$\begin{aligned} &x^2[2(2m+1)(m-1) - x^2]j_m(Kx)j_m(x) + 4Kx(m+2(m-1)j_{m+1}(Kx)j_{m+1}(x) + 4Kx[x^2 - \\ &(m+1)(m-1)(m+2)]j_{m+1}(Kx)j_{m+1}(x) + 2x[x^2 - 2m(m-1)(m+2)]j_{m+1}(Kx)j_{m+1}(x) = 0 \end{aligned} \tag{10}$$

where $x = k_2R$, $K = {k_1}/{k_2}$,$k_2 = {2\pi fR}/{C_2}$,$k_1 = {2\pi fR}/{C_1}$. R and f are the radius and natural frequency, respectively.