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# An induction-based magnetohydrodynamic 3D code for finite magnetic Reynolds number liquid-metal flows in fusion blankets

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## HIGHLIGHTS

- A new induction-based magnetohydrodynamic code was developed using a finite difference method.
- The code was benchmarked against purely hydrodynamic and MHD flows for low and finite magnetic Reynolds number.
- Possible applications of the new code include liquid-metal MHD flows in the breeder blanket during unsteady events in the plasma.

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## ABSTRACT

Most numerical analysis performed in the past for MHD flows in liquid-metal blankets were based on the assumption of low magnetic Reynolds number and involved numerical codes that utilized electric potential as the main electromagnetic variable. One limitation of this approach is that such codes cannot be applied to truly unsteady processes, for example, MHD flows of liquid-metal breeder/coolant during unsteady events in plasma, such as major plasma disruptions, edge-localized modes and vertical displacements, when changes in plasmas occur at millisecond timescales. Our newly developed code MOONS (Magnetohydrodynamic Object-Oriented Numerical Solver) uses the magnetic field as the main electromagnetic variable to relax the limitations of the low magnetic Reynolds number approximation for more realistic fusion reactor environments. The new code, written in Fortran, implements a 3D finite-difference method and is capable of simulating multi-material domains. The constrained transport method was implemented to evolve the magnetic field in time and assure that the magnetic field remains solenoidal within machine accuracy at every time step. Various verification tests have been performed including purely hydrodynamic flows and MHD flows at low and finite magnetic Reynolds numbers. Test results have demonstrated very good accuracy against known analytic solutions and other numerical data.

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## 1. Introduction

Magnetohydrodynamic (MHD) codes are important tools for design and analysis of liquid-metal blankets [1]. Induced magnetic fields are often dismissed as negligible compared to external magnetic fields in many codes used to simulate liquid-metal MHD flows in fusion reactor environments, but this is not always the case. Major plasma disruptions, edge-localized modes and vertical displacements may result in strong electromagnetic interactions in the liquid-metal, which cannot be described with the inductionless approximation. In addition, magnetic Reynolds number ($Re_m$) based on large duct lengths (e.g. long poloidal "banana" segments) may yield moderate values comparable with unity.

The magnetic induction (B) formulation, based on utilization of the magnetic field as the main electromagnetic variable, is more general and has several advantages over the electric potential ($\varphi$) based formulation. Induced magnetic fields and its transport are captured in the B-formulation but ignored in the $\varphi$-formulation. Another advantage of the B-formulation is that conservation of charge, a challenging and important constraint in MHD computations [2], is automatically enforced if the magnetic field is numerically solenoidal. The goal of this paper is to introduce a new induction-based code for finite $Re_m$ MHD flows and present test results and its capabilities.

## 2. Mathematical formulation

Combining Ohm's law, Faraday's law, and the Ampère–Maxwell equation yields the induction equation. Momentum, continuity and induction equations are non-dimensionalized by characteristic

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velocity, $U$, length, $L$, time, $L/U$, pressure, $\rho U^2$, and magnetic field, $B$. Assuming incompressible and isothermal conditions, the dimensionless momentum, induction, mass continuity and magnetic field continuity equations are:

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \bullet (\mathbf{uu}) = -\nabla p + \frac{1}{Re}\nabla^2 \mathbf{u} + \frac{Ha^2}{ReRe_m}(\nabla \times \mathbf{B}^{ind}) \times (\mathbf{B}^0 + \mathbf{B}^{ind}), \quad (1)$$

$$\frac{\partial(\mathbf{B}^0 + \mathbf{B}^{ind})}{\partial t} - \nabla \times \{\mathbf{u} \times (\mathbf{B}^0 + \mathbf{B}^{ind})\} + \frac{1}{Re_m}\nabla \times \left\{ \frac{1}{\bar{\sigma}} \nabla \times \left( \frac{\mathbf{B}^0 + \mathbf{B}^{ind}}{\bar{\mu}_m} \right) \right\} = 0, \quad (2)$$

$$\nabla \bullet \mathbf{u} = 0, \quad (3)$$

$$\nabla \bullet \mathbf{B} = 0. \quad (4)$$

Here, $\mathbf{u}$, $p$, $\mathbf{B}$ are dimensionless velocity, pressure and magnetic field (separated into induced, $\mathbf{B}^{ind}$, and applied, $\mathbf{B}^0$) respectively. In Eqs. (1) and (2), Reynolds number ($Re$) Hartmann ($Ha$) magnetic Reynolds number ($Re_m$) and dimensionless ratios of electrical conductivity and magnetic permeability are defined as

$$Re = \frac{UL}{\nu}, \quad Ha = BL\sqrt{\frac{\sigma}{\rho\nu}}, \quad Re_m = \mu_m \sigma UL,$$

$$\bar{\sigma} = \begin{cases} 1 & \text{fluid} \\ \dfrac{\sigma_w}{\sigma} & \text{wall} \end{cases}, \quad \bar{\mu}_m = \begin{cases} 1 & \text{fluid} \\ \dfrac{\mu_{m,w}}{\mu_m} & \text{wall} \end{cases}.$$

Here, $\nu$, $\rho$, $\sigma$, $\mu_m$ are kinematic viscosity, density, electrical conductivity and magnetic permeability of the fluid while $\sigma_w$ and $\mu_{m,w}$ pertain to the wall respectively. Reynolds number is the ratio of inertial to viscous forces. Hartmann number squared is the ratio of electromagnetic to viscous forces. Magnetic Reynolds number is the ratio of magnetic field convection to diffusion. In this study $\bar{\mu}_m$ is assumed to be unity.

Similar to Eq. (3), Eq. (4) is a physical constraint on the magnetic field to remain solenoidal. Mathematically, Eq. (4) does not need to be solved along with Eq. (2) but this may lead to unphysical forces in the momentum equation [3,4]. The constrained transport (CT) method is used to enforce Eq. (4) and will be discussed in Section 3.

All solved variables begin with zero magnitude. Typical boundary conditions (BC) for velocity and pressure are Dirichlet, Neumann and periodic. The most physically reliable BCs for the magnetic field is $\mathbf{B} = 0$ far from the flow domain. In practice, this can require many grid points and be computationally expensive. Several methods can approximate or reconstruct this BC. First, the pseudo-vacuum BC, expressed as

$$\frac{\partial B_{normal}}{\partial n} = 0, \quad B_{tangential} = 0, \quad (5)$$

can be applied at the interface between the flow-containing wall and the non-conducting exterior (vacuum). Second, a decay function of the form $\mathbf{B} \propto r^{-n}$ can be used where $r$ and $n$ are the distance from the flow domain and a decay parameter. Third, the Boundary Element Method solves for the magnetic field using a magnetic scalar potential, obtained from a Laplace equation by applying the magnetic field curl-free property outside the flow domain and the divergence-free constraint [5].

In this study we use the pseudo-vacuum BCs as this approach was successfully utilized in commercial solvers ANSYS and FLUENT and can produce physically realistic results for a wide range of MHD problems [6–8].

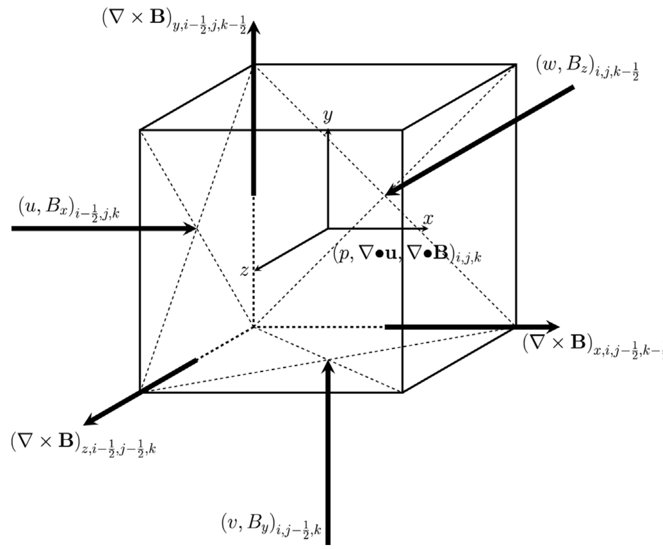


**Fig. 1.** Staggered variables on computational cell.

## 3. Numerical procedure

Our code, MOONS, solves the governing Eqs. (1)–(4) in rectilinear coordinates. Second order accurate finite-difference schemes are used to approximate all spatial derivatives on a staggered grid (Fig. 1). Centered difference stencils were used to compute derivatives, including the advection term in Eq. (1). Non-uniform grids are generated using Robert's stretching functions defined in [9], which ensure that a sufficient number of cells are present in boundary layers. Momentum and induction equations are solved separately at each time level, where first order accurate explicit time marching is used for all terms except pressure. Pressure is treated purely implicitly. A projection method is used to enforce a divergence-free velocity field, where Gauss-Seidel method is used to iteratively solve the pressure Poisson equation [10]. Diagonal Preconditioned Conjugate Gradient method was used in place of Gauss-Seidel for Shercliff and Hunt flows.

The CT method, described in Ref. [3], is implemented and enforces Eq. (4) within machine accuracy at every time step as long BCs are compatible with Eq. (4) and initial conditions satisfy Eq. (4). The main idea is that $\mathbf{B}$ and $\nabla \times \mathbf{B}$ are staggered on cell faces and edges respectively (Fig. 1), resulting in perfect numerical cancellation when computing Eq. (4). Although not shown in Fig. 1, $\mathbf{u} \times \mathbf{B}$ is also located on cell edges. Interpolations of variables between different cell locations are performed with second order accuracy.

## 4. Verification test cases

Several verification tests were conducted including: (1) purely hydrodynamic flows, (2) MHD flows at low $Re_m$ and (3) MHD flows at finite $Re_m$. The goal of the purely hydrodynamic verification tests was to address grid refinement, spatial order of accuracy and benchmark the hydrodynamic component of MOONS against available numerical solutions.

### 4.1. Hydrodynamic lid-driven cavity flow

A lid-driven cavity flow is a classic benchmark for fluid dynamic codes due to the complex flow features including transition to

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