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Fluid filling of a membrane tube with self-weight

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A B S T R A C T

The filling of a membrane tube with self-weight by a fluid is studied theoretically for the first time. The fundamental problem depends on two non-dimensional parameters, β and γ which represent the importance of internal pressure and membrane self-weight respectively. The nonlinear equations are solved by an efficient numerical integration method. It is found that membrane self-weight has considerable effect on the tube geometry and the tension of the membrane at low filling pressures.

1. Introduction

Due to their ease of set up, dismantling, and transport, membrane tubes filled with fluids or solids are important as temporary barriers for flood control and pollution containment (e.g. Lawson, 2008; Pilarczyk, 2000).

The fluid-filled, inextensible membrane tube of negligible self-weight resting on a flat solid foundation was first analysed by Demiray and Levinson (1972). The solution was expressed in terms of elliptic functions which are difficult to evaluate and the resulting cross sections are somewhat inaccurate. Wang and Watson (1981) also found the elliptic function solution and noted the problem is analogous to a clamped elastica rod. Aside from numerical integration, Wang and Watson (1981) developed asymptotic formulas which are easier to use. Leshchinsky et al. (1996) noted that theory and experiments with water-filled tubes agree superbly. Approximate and numerical methods of the exact solution were introduced by Cantre (2002) and Guo et al. (2014a,b). Namias (1985) added a top loaded plate. The case when the tube is floating on an exterior liquid was considered by Szyszkowski and Glockner (1987). Plaut and Suherman (1998) studied the case when the bottom foundation is deformable, and also with external liquid on one side. The case when the tube is filled with different liquids was investigated by Antman and Shagerl (2005) and Malik and Sysala (2011). The related problem of a membrane strip holding liquid but fixed at two edges (and with impounding) was considered by Ghavanloo and Daneshmand (2010). Some work has also been done for a membrane strip holding a liquid and hanging by fixed edges (Wang, 1982).

The present paper considers the membrane tube with self-weight and filled with a fluid. Unlike the previous studies, where either membrane self-weight or hydrostatic pressure is absent, this basic problem has no exact solution. We shall use an efficient numerical method to study its properties.

2. Formulation

Fig. 1a shows the cross section of a fluid-filled membrane tube of total perimeter length L. Due to gravity, the tube flattens on the foundation with a contact length of cL ($c < 0.5$). Let the Cartesian coordinate axes (x', y') be located at one end of the contact length as shown. The hydrostatic pressure p, whether for liquid or gas, at any point is

$$p = p_0 - p_a - \rho g y' \tag{1}$$

Here p_0 is the pressure at the bottom, p_a is the ambient pressure, ρ is the fluid density, and g is the gravitational acceleration. Fig. 1b shows an elemental segment ds' where s' is the arc length from the origin. Let the angle of inclination of the lower end of the segment with the horizontal be θ and the tension there be T'. The self-weight of this segment is wds' where w is the weight per area of the membrane. The force balance along the segment is

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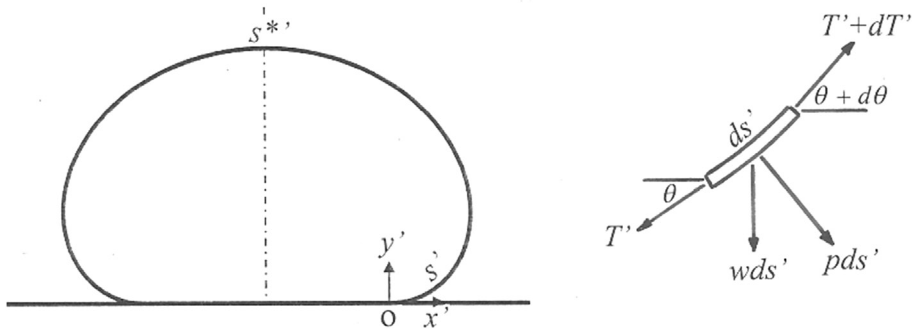

Fig. 1. (a) Cross section of a membrane tube. Origin of computation is at O and ends at s^* on top. (b) Force balance on an elemental segment.

$$T' + dT' = T' + wds' \sin\theta \tag{2}$$

The force balance normal to the segment is

$$T'd\theta = pds' + wds' \cos\theta \tag{3}$$

Normalize all lengths by L, the tension by $\rho g L^2$ and drop primes. Eqs.(2) and (3) become

$$\frac{dT}{ds} = \gamma \sin\theta \tag{4}$$

$$T\frac{d\theta}{ds} = \beta - y + \gamma \cos\theta \tag{5}$$

Here

$$\beta = \frac{p_0 - p_a}{\rho g L}, \quad \gamma = \frac{w}{\rho g L} \tag{6}$$

are important non-dimensional parameters: β represents the importance of internal pressure, and γ represents the importance of membrane self-weight. The kinematic equations are

$$\frac{dx}{ds} = \cos\theta, \quad \frac{dy}{ds} = \sin\theta \tag{7}$$

The boundary conditions are that at the origin

$$x(0) = 0, \quad y(0) = 0, \quad \theta(0) = 0 \tag{8}$$

and at the top of the membrane at $s^* = (1 - c)/2$

$$\theta(s^*) = \pi, \quad x(s^*) = -\frac{c}{2} = s^* - \frac{1}{2} \tag{9}$$

Eqs. (4), (5), (7)–(9) are to be solved.

When $\gamma = 0$, the problem reduces to the fluid-filled membrane tube without self-weight. When γ, β are large, the problem tends to the membrane tube with constant internal pressure. Since tension T is positive and θ is increasing, Eq. (5) shows that

$$y \le \beta - \gamma \tag{10}$$

This means the tube would collapse completely when $\gamma \ge \beta$ or when the pressure difference could not support the weight of the tube material.

3. Numerical solution

For our problem, no exact solution exists. Eqs. (5) and (7) is fourth order, plus unknown s^*. There are five boundary conditions Eqs.(8) and (9). This difficult problem can be solved by the following efficient numerical one-dimensional shooting method. We guess the tension at the origin $T(0)$ and integrate Eqs.(5) and (7) as an initial value problem. In the mean time the error squared E is defined

$$E = \left[x(s) + \frac{1}{2} - s\right]^2 + [\theta(s) - \pi]^2 \tag{11}$$

When E becomes less than a threshold, say 10^{-9}, the integration is terminated and we set $s = s^*$. The contact length is $c = 1{-}2s^*$, the height is $y^* = y(s^*)$. Since we started with $T(0)$ and from Eq. (4) $dT/ds > 0$, thus the maximum tension increases monotonically to $T^* = T(s^*)$ at the top of the membrane. The volume per depth (cross sectional area) enclosed is

$$V^* = 2\int_0^{y^*}\left(x + \frac{c}{2}\right)dy = 2\int_0^{s^*}\left[x(s) + \frac{c}{2}\right]\sin\theta ds \tag{12}$$

Eq. (12) is equivalent to differential equation

$$\frac{dV}{ds} = 2\left[x(s) + \frac{c}{2}\right]\sin[\theta(s)], \quad V(0) = 0 \tag{13}$$

Then $V^* = V(s^*)$. Table 1 shows our computed results for various β and γ values. Notice $\gamma < \beta$ to prevent collapse. Important design parameters are the maximum height y^*, the volume V^* and the maximum tension T^* (the dimensional tension is $T^*\rho g L^2$). The $\gamma = 0$ (no membrane self-weight) results agree qualitatively with those of Demiray and Levinson (1972) and Wang and Watson (1981), since no numerical values have been published until now.

Fig. 2 shows the tube cross-sections for given internal pressure parameter β and various membrane self-weight parameter γ. It is seen that the membrane self-weight has considerable effect on the geometry.

We also note that for small β most of the tube is almost flat, and for large β and low γ the tube is almost circular.

4. Effect of self-weight

The effect of self-weight is considerable, especially at low internal pressures. Fig. 3a (adapted from Table 1) shows the height of the tube y^* versus the self-weight parameter γ for various internal pressure parameters β. For a given membrane tube and a given filling material, one can determine γ. Then as internal pressure is increased, the corresponding tube height can be read from the abscissa. The figure also shows the relative error boundaries if we ignore the self-weight effect. For example, if $\gamma = 0.1$ the error for ignoring self-weight is over 20% when $\beta = 0.5$, and the error is over 10% when pressure is increased to $\beta = 1$. The error is about 2%