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Elasto-plastic analysis of the surrounding rock mass in circular tunnel based on the generalized nonlinear unified strength theory

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A B S T R A C T

The present paper aims to establish a versatile strength theory suitable for elasto-plastic analysis of underground tunnel surrounding rock. In order to analyze the effects of intermediate principal stress and the rock properties on its deformation and failure of rock mass, the generalized nonlinear unified strength theory and elasto-plastic mechanics are used to deduce analytic solution of the radius and stress of tunnel plastic zone and the periphery displacement of tunnel under uniform ground stress field. The results show that: intermediate principal stress coefficient *b* has significant effect on the plastic range, the magnitude of stress and surrounding rock pressure. Then, the results are compared with the unified strength criterion solution and Mohr–Coulomb criterion solution, and concluded that the generalized nonlinear unified strength criterion is more applicable to elasto-plastic analysis of underground tunnel surrounding rock.

1. Introduction

The original equilibrium state of stress is disrupted by underground engineering excavation, which causes the stress to be redistributed [1–3]. The plastic zone may appear when surrounding rock stress exceeds the yield stress. The size and shape of plastic zone are important for stability analysis of tunnel surrounding rock and quantitative design of the support.

Zhang et al. studied the secondary stress state and plastic radius of tunnel surrounding rock by Mohr–Coulomb strength criterion (hereinafter referred to as MC criterion), while Zeng et al. carried out the same research based on Hoek–Brown strength criterion (hereinafter referred to as HB criterion) [4–7]. Although the studies had certain values of theory and practice, HB and MC criteria ignored the effects of intermediate principal stress on rock mass strength, which led to inaccurate results. Considering the effect of different degrees of intermediate principal stress on the yield of surrounding rock, Zhang et al. derived the analytical solutions of elasto-plastic zone stress and plastic zone radius based on Drucker–Prager strength criterion (hereinafter referred to as DP criterion) [8,9]. However, DP criterion could not distinguish the difference between tensile and compressive meridian, which led to the fact that the results were inconsistent with the rock and soil triaxial test data; Hu et al. applied the unified strength criterion to deduce a boundary equation of plastic zone in the tunnel surrounding rock under non-uniform stress field [10]. However, the unified strength criterion is a combination of the linear criterion [11]. It cannot unify the nonlinear criterion, such as Huber–von Mises criterion, but only approximate it, which determines the unified strength criterion not apply to most rock materials.

Based on the unified strength theory and twin-shear models, Zan et al. revised HB strength criterion and put forward the rock nonlinear unified strength theory; in order to combine the unified strength theory with nonlinear unified strength theory, they proposed generalized nonlinear unified strength theory with wider application range [12,13]. As shown in Fig. 1, the generalized nonlinear unified strength theory includes not only several well-known strength criteria, but also many new strength criteria, therefore it has extensive applicability.

Considering the characteristics of rock and the effect of intermediate principal stress on rock strength, this paper applied the generalized nonlinear unified strength criterion and elasto-plastic mechanics for deducing the analytical solutions of elasto-plastic zone stress, plastic zone radius and displacement under uniform stress field.

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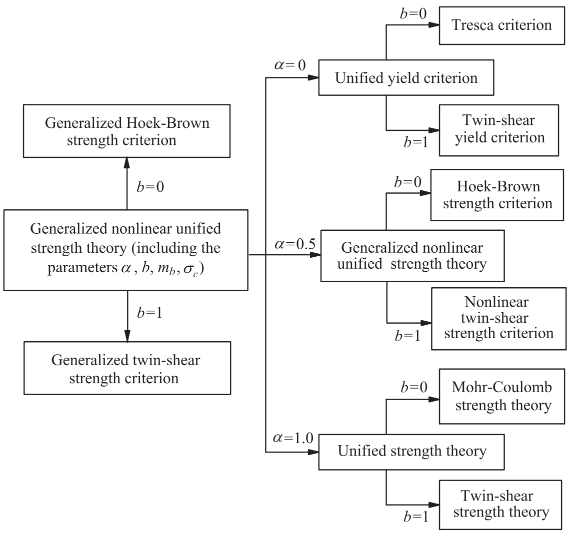

Fig. 1. Relationship between the generalized nonlinear unified strength theory and other strength theories.

2. Generalized nonlinear unified strength theory of rock

In 1980, based on the results of research into the brittle failure of intact rock and on model studies of jointed rock mass behavior, Hoek and Brown proposed Hoek–Brown strength criterion [14]. The criterion started from the properties of intact rock and then introduced factors to reduce these properties on the basis of the characteristics of joints in a rock mass. In 1992, Hoek et al. modified HB strength criterion so that it can be applied to both rock and rock mass, namely the generalized HB rock strength criterion [15]. The criterion exactly reflects the nonlinear properties of rock and could explain the influence of tensile stress zone, low stress area and minimum principal stress on rock strength. However, it ignores the effect of intermediate principal stress on rock mass failure, which is the critical shortcoming.

Yu proposed a generalized twin-shear strength theory, on the basis of which he put forward a unified strength theory [16]. Since it considered all stress components and different effects of them on material yield and failure, the theory is suitable for the geotechnical materials with different yield strength in tension and compression and reflects the effect of the intermediate principal stress. However, the unified strength theory can neither completely fit rock nonlinear strength criteria, nor reflect the structural characteristics or the damage degree of structure, which leads to a certain deviation compared with actual results.

Based on the mathematical expressions of generalized HB strength criterion and the twin-shear model, Yu et al. established the generalized nonlinear unified strength theory which is suitable for the strength characteristics of both soft and hard rocks under triaxial stress state [17]. The serial limit surfaces of the generalized nonlinear unified strength theory cover the whole convex region, and encompass the generalized Hoek–Brown strength criterion and the generalized nonlinear twin-shear strength criterion as two special cases. Yield function F of the generalized nonlinear unified strength criterion is expressed as:

$\sigma_2 \leqslant \dfrac{\sigma_1 + b\sigma_3}{1+b}$,

$$F = \left[\sigma_1 - \frac{1}{1+b}(b\sigma_2 + \sigma_3)\right] - \sigma_c\left[\frac{m}{(1+b)\sigma_c}(b\sigma_2 + \sigma_3) + s\right]^{\alpha} = 0 \tag{1}$$

When $\sigma_2 > \frac{\sigma_1 + b\sigma_3}{1+b}$,

$$F = \left[\frac{1}{1+b}(\sigma_1 + b\sigma_2) - \sigma_3\right] - \sigma_c\left[\frac{m_b}{\sigma_c}\sigma_3 + s\right]^{\alpha} = 0 \tag{2}$$

where σ_1, σ_2, σ_3 are three principal stresses under the failure condition; b the intermediate principal stress coefficient, and the larger the value of b is, the stronger the effects of intermediate principal stress is; σ_c the uniaxial compressive strength of the intact rock material; and m_b the reduced value of the material constant m_i and is given by Hoek et al. [18].

$$m_b = m_i \exp\left[\frac{GSI - 100}{28 - 14D}\right] \tag{3}$$

s and α are parameters for the rock mass given by the following relationships.

$$s = \exp\left[\frac{GSI - 100}{9 - 3D}\right] \tag{4}$$

$$\alpha = \frac{1}{2} + \frac{1}{6}\left[\exp\left[-\frac{GSI}{15}\right] - \exp\left[-\frac{20}{3}\right]\right] \tag{5}$$

where GSI is a geological strength index; and D a factor which depends on the degree of disturbance of rock mass subjected by blast damage and stress relaxation. It varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses.

3. Elasto-plastic analysis under axisymmetric stress condition

3.1. Mechanical model

Mechanical model of tunnel surrounding rock after excavation is shown in Fig. 2. Given the following conditions:

(1) Surrounding rock is homogeneous, continuous, and isotropic.
(2) It can be simplified as plane strain problem, provided that circular tunnel is of sufficient buried depth and length.
(3) Ignoring the effects of the surrounding rock weight on the yield, tunnel is under uniform stress field (p_0), while the support system provides uniform support resistance (p_i).
(4) The radius of the plastic zone is R_p after tunnel excavation.

3.2. Analysis of stress state

The elasto-plastic mechanical analysis of circular tunnel rock under static hydraulic pressure can be considered as plane strain problems. Since σ_r, σ_θ, σ_z are mutually orthogonal, they can be

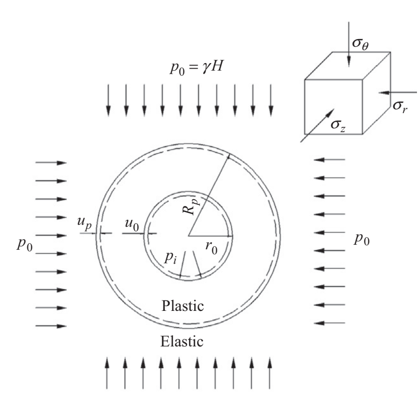

Fig. 2. Mechanical model of tunnel surrounding rock.