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Analysis of the stress ratio of anisotropic rocks in uniaxial tests

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ABSTRACT

The effect of structural discontinuities on the progressive failure process of anisotropic rocks should be paid particular attention. The crack damage stress σ_{cd}, also considered as the yield strength, and the relationship between σ_{cd} and the uniaxial peak strength σ_{ucs} of anisotropic rocks for different orientations θ of the isotropy planes with respect to the loading directions were investigated theoretically and experimentally. A theoretical relation of σ_{cd}/σ_{ucs} with the function of the shape parameter m was established. Additionally, uniaxial compression tests of shale samples were conducted for several inclinations θ. The test result of σ_{cd}/σ_{ucs} was close to the theoretical value for a given orientation. Furthermore, both experimental results and theoretical solutions of σ_{cd}/σ_{ucs} were independent of the inclination θ while σ_{cd} and σ_{ucs} were strongly affected by θ. The strength ratio σ_{cd}/σ_{ucs} may therefore be an intrinsic property of anisotropic rocks and could be used to predict the failure of rock samples.

1. Introduction

Many underground projects, such as underground excavation and borehole drilling, are located in anisotropic rocks [1,2]. The disturbance caused during the excavation and drilling process generates stress redistribution and concentration, which in turn results in deformations and development of fractures in the rock mass [3,4]. Knowledge of the deformation and failure process of the rocks is of the utmost importance in the assessment of the stability of an underground cavity.

Anisotropic rocks are heterogeneous and discontinuous media, containing defects such as cracks, pores, joints, and especially bedding or foliation. These pre-existing micro-discontinuities play an important role in the progressive failure process of the rocks. Under external compressive load, the micro-discontinuities may produce local tensile stresses that lead to the initiation, propagation, nucleation, and interaction of micro cracks and finally rock failure [3,5]. It is commonly accepted that uniaxial compression stress-strain responses can be split into several stages using different stress thresholds that are important to the understanding of the deformation-failure of brittle rock [6–8]. In particular, the crack damage stress σ_{cd} is considered as the yield strength of a rock specimen, above which some physical properties, such as acoustic emissions, permeability, and wave velocity, have large responses [9–11]. It is thus meaningful to study the correlation between the crack damage stress and the failure stress of rock.

Much research has been conducted in an attempt to establish a theoretical quantitative correlation between the crack damage stress σ_{cd} and the failure stress σ_{ucs} of rock. Wu et al. studied the relationship between the elastic ultimate strength (the crack damage stress) and the failure strength using a Weibull distribution statistic model [12]. Cao et al., Yang et al. and Chen et al. derived quantitative relations between the peak stress and the average stress based on the damage softening statistical constitutive model for rocks [13–15]. Qin et al. investigated stress instability criteria in a shear state with the introduction of one-dimensional renormalization group (RG) theory [16]. Xue et al. introduced two-dimensional RG theory to derive a theoretical expression of σ_{cd}/σ_{ucs} based on the solutions of $\varepsilon_{cd}/\varepsilon_{ucs}$ [9,17].

The present paper introduces one-dimensional RG theory to derive directly the relation of σ_{cd} and σ_{ucs}. The structure of the remainder of paper is as follows. Section 2 gives a detailed mathematical derivation of the theoretical solution of σ_{cd}/σ_{ucs}. Section 3 presents experimental results and a comparison with theoretical solutions. Finally, conclusions are presented in Section 4.

2. Theoretical study

Because of the grain-scale heterogeneity, it is assumed that the strength of each individual cell obeys the Weibull distribution

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[18,19]. Thus, to characterize the local failure strength, we adopt the probability density function:

$$f(\sigma)=\frac{m}{\sigma_0}\left(\frac{\sigma}{\sigma_0}\right)^{m-1}\exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{m}\right] \tag{1}$$

And then the broken probability function is

$$P(\sigma)=1-exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{m}\right] \tag{2}$$

where σ is the cell stress; σ_0 the mean strength of the cell; and m a shape parameter that characterizes the degree of homogeneity in the rock.

Introducing the one-dimensional RG theory to the above functions, Qin et al. derived the critical broken probability, which is equivalent to the concept of the crack damage stress σ_{cd} [20].

$$P_{cd}=1-0.5^{\frac{1}{2^m-1}} \tag{3}$$

Substitution of Eqs. (3) into (2) yields

$$\frac{\sigma_{cd}}{\sigma_0}=\left(\frac{\ln 2}{2^m-1}\right)^{\frac{1}{m}} \tag{4}$$

The load capacity probability function of a rock specimen can be written as

$$P=\sigma\exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{m}\right] \tag{5}$$

The first-order derivative of Eq. (5) is

$$\frac{\sigma_{ucs}}{\sigma_0}=\left(\frac{1}{m}\right)^{\frac{1}{m}} \tag{6}$$

And then, taking the ratio of Eqs. (4) and (6) gives

$$\frac{\sigma_{cd}}{\sigma_{ucs}}=\left(\frac{m\ln 2}{2^m-1}\right)^{\frac{1}{m}} \tag{7}$$

The quantitative relationship between the crack damage stress σ_{cd} and the failure strength σ_{ucs} of rock is thus established. It is found that the ratio of σ_{cd}/σ_{ucs} is a function of m, which is used to evaluate the discreteness of the rock's strength.

Employing statistics damage theory and Hooke's law, researchers have derived the formula for m under uniaxial compression [14,15]:

$$m=\frac{1}{\ln\left(\frac{E\varepsilon_{ucs}}{\sigma_{ucs}}\right)} \tag{8}$$

where E is the elastic modulus; and ε_{ucs} the axial peak strain. Rearranging Eq. (8) gives

$$m=\frac{1}{\ln(E)-\ln(E_s)} \tag{9}$$

where E_s is the secant modulus passing through the peak strength point.

Thus, m is a function of the elastic modulus and the secant modulus through the peak point of the rock uniaxial compression stress–strain curve. It represents the degree of difference between E and E_s. It is believed that rock is more homogeneous and brittle when m is larger.

To determine the theoretical solution of σ_{cd}/σ_{ucs}, we first calculate m, which is determined by the peak strength σ_{ucs}, the axial peak stain ε_{ucs}, and the modulus of elasticity E in the uniaxial compressive loading test.

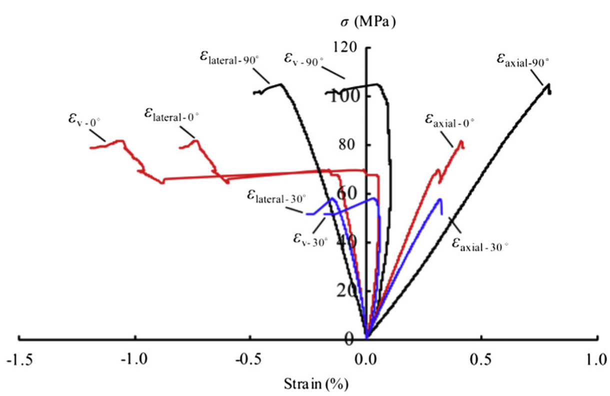
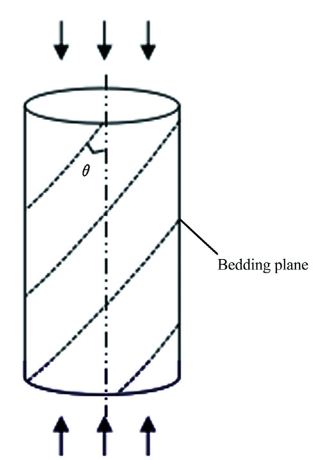
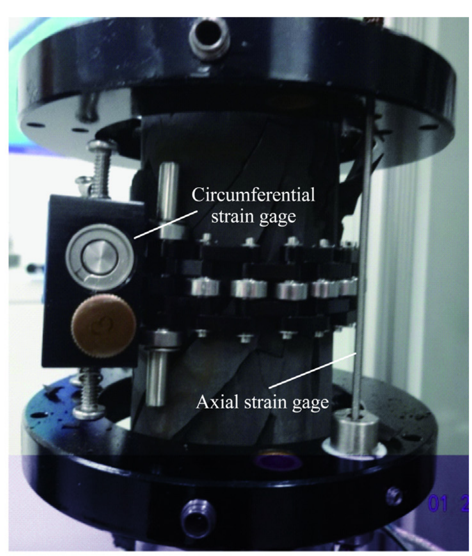


Fig. 2. Stress–strain curves for shale specimens with orientations θ = 0°, 30°and 90°.



(a) Illustration of a shale sample and its bedding plane relative to the loading orientation



(b) Positions of strain gauges in uniaxial compression tests

Fig. 1. Illustration of shale sample and testing apparatus.