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A fuzzy set plasticity model for cyclic loading of granular soils

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Abstract

A constitutive model for describing the stress–strain behavior of granular soils subjected to cyclic loading is presented. The model is formulated using fuzzy set plasticity theory within the classical incremental plasticity theory framework. A special membership function is introduced to provide an analytical and simple geometrical interpretation to formulate hardening, hysteresis feature, material memory, and kinematic mechanisms without resorting to complicated kinematic hardening formulations. The model can accurately describe cyclic loading, dilatancy, material theory and critical state soil mechanics features effects. Two series of cyclic drained triaxial tests data are considered. The characteristic features of behavior in granular soils subjected to cyclic loading are captured.

Keywords: Constitutive model; Fuzzy set; Plasticity; Cyclic loading; Hardening hysteresis feature

1. Introduction

Cyclic response of granular materials is complex due to the pressure and specific volume dependency of the stress–strain relationship and the highly nonlinear behavior of the soil matrix. Until now, the mechanical behavior of granular soils has been mainly represented with constitutive models which need different sets of constitutive parameters for each density and effective confining pressure. In fact, the study of loading and unloading response in granular soils and development of relationships for its prediction in natural formations and engineered materials has been a major area of research in modern geomechanics. Concerted effort has been made to develop predictive capabilities associated with topics such as earthquake engineering, soil-structure interaction, soil liquefaction, off-shore engineering, etc.

Development of constitutive models for a wide range of engineering materials, including soils, has been found extensively for recent decades [1–15]. A majority of the models is based on the incremental plasticity theory. Within the framework of classical plasticity theory, isotropic hardening has been proved sufficient to simulate the stress–strain response of soil subjected to monotonic loading while kinematic hardening and mixed hardening has been typically used to mimic hysteretic phenomena of soil under cyclic loading.

Nowadays, the cyclic behavior of unbound granular materials under traffic loading is another challenging task for geotechnical engineers. A typical example is railroad ballast. Thus, it is of special interest to determine the over characteristics and constitutive properties of the ballast and to ensure stable and long-lasting properties for such a material that is not homogeneous. The research focuses on the development of a cyclic constitutive model based

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on fuzzy set concepts, its numerical integration, and finite element implementation as well.

Unlike convention elasto-plastic hardening models, the fuzzy set model is physically intuitive and easy to visualize. It provides analytical and simple geometrical interpretations to formulate hardening, hysteresis features, material memory, and kinematic mechanisms. In this model, based on fuzzy set plasticity theory, the basic concept rests on the assumption that there exists a fuzzy surface which in many ways resemble a bounding surface. At each point within the fuzzy surface, the value of plastic hardening moduli is defined by the membership function. In this view, Bao et al. presented a transparent and accurate kinematic-cyclic constitutive model to capture the important features of volume change and pore water pressure build-up related to soil cyclic mobility [16].

In this study, a cyclic plasticity model based on fuzzy plasticity theory is presented to model the cyclic behavior of unbound granular materials under repeated loads. The enhanced fuzzy-set model is built to adapt the simply format equations of plastic moduli and plastic potential to simulated the pavement materials deformation problems particularly related to cyclic mobility. Two series of cyclic drained triaxial tests data are considered. The characteristic features of behavior in granular soils subjected to cyclic loading are captured.

2. Preliminaries

2.1. Notation

In the model presented, the material behavior is assumed isotropic and rate independent in both elastic nad elastic–plastic response. Compression is considered positive and tension is negative. For simplicity, triaxial stress notation $p' - q$ is adopted throughout; $p' = (\sigma_1' + 2\sigma_3')/3$ is the mean effective stress and $q = \sigma_1' - \sigma_3'$ is the deviator stress, where σ_1' and σ_3' are the axial and radial stresses, respectively. The corresponding work conjugates are volumetric strain $\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$ and deviatoric strain $\varepsilon_d = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$. The pairs of stresses and strains are abbreviated in the vector form as $\boldsymbol{\sigma}_1' = [p', q]^T$ and $\boldsymbol{\epsilon} = [\varepsilon_v, \varepsilon_d]^T$. The total strain rate is decomposed into elastic and plastic parts according to

$$\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}^{\boldsymbol{e}} + \dot{\boldsymbol{\epsilon}}^{\boldsymbol{p}} \tag{1}$$

where a superimposed dot indicates an increment, and the superscripts $\boldsymbol{e}$ and $\boldsymbol{p}$ denote the elastic and plastic components, respectively.

2.2. Elastic behavior

The tangential elastic moduli are calculated assuming that the slope of unloading/reloading occurs along a κ line in the $e - lnp'$ plane. The moduli are then defined as

$$K = \frac{(1+e)p'}{\kappa}, \quad G = \frac{3(1-2v)}{2(1+v)} \frac{(1+e)p'}{\kappa} \tag{2}$$

where e is the void ratio and v is the Possion's ratio.

2.3. Membership function

The membership function has been involved in the plastic modulus equations. When $\gamma = 1$, the material behaves purely elastically and the corresponding value of the plastic modulus is infinite, while when $\gamma = 0$, the material reaches a fully plastic state and the plastic modulus is equal to the value on the fuzzy surface, i.e. $H = H^*$.

With the assistance of the membership function γ, we can readily construct reversal plastic loading without resorting to a kinematic hardening rule. The basic rules of kinematic mechanism for the membership function are:

- Plastic loading: $\dot{\gamma} < 0$
- Plastic unloading: $\dot{\gamma} < 0$
- Elastic loading: $\dot{\gamma} \geqslant 0$
- Elastic unloading: $\dot{\gamma} \geqslant 0$

Although the value of the membership function is 1 at a fully elastic state and 0 at the fully plastic state, the assignment of the value in elastoplastic state is deterministic and can be arbitrarily defined as needed. A linear variation with respect to stress state was adopted in this study.

Fig. 1 displays an example of the deviatoric stress–strain response and evolution of the membership function for a material subjected to two varied amplitude cyclic loading under a conventional triaxial stress path. The unloading–reloading points take place in two different stress levels $q = 156$ kPa and $q = 231$ kPa, respectively. The two graphs on the left highlight cycle 1 with the loading from 0 to 156 kPa and unloading from 156 to 0 kPa (in solid line). The other two graphs highlight the cycle 2 with the

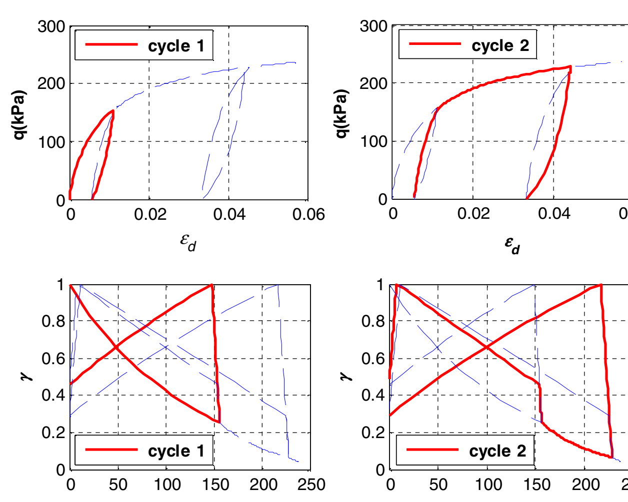

Fig. 1. Deviatoric stress–strain curve and evolution of the membership function γ for cycle 1 and cycle 2.