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Elastoplastic and contact analysis based on consistent dynamic formulation of co-rotational planar elements

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ABSTRACT

Co-rotational formulation is one of the approaches applicable to geometrically nonlinear problems in structural analysis. Based on the assumptions of small degree of strain and large displacement, the CR formulation allows an accurate geometrically nonlinear structural analysis. In this paper, an improved CR formulation for dynamic nonlinear analysis is developed for the planar element. Also, extension of the local formulation is conducted for an elastoplastic problem. A simplified approach for planar elastoplastic analyses is proposed by maintaining the original feature of the CR formulation. Moreover, a modified approach in the local formulation is suggested in order to improve the level of accuracy in the plastic state. Contact analysis using the Lagrange multiplier is then implemented. During the present validation procedure of the proposed approaches, four different planar elements are employed. Those elements are investigated with regard to efficiency and accuracy in a general geometrically nonlinear and a geometrically nonlinear/elastoplastic problem, respectively. It is found that the present dynamic formulations gives both accuracy and consistency for all types of the plane element. Also, comparison regarding the choice of the local formulation for the elastoplastic problem is systematically conducted and demonstrated.

1. Introduction

Structures used in many engineering applications exhibit geometrically and materially non-linear characteristics. Such characteristics are among the most important physical aspects in the fields of the mechanical and aerospace engineering. Especially in the aerospace engineering field, structures encounter high pressure and temperature boundary conditions, specifically the structures of high-speed vehicles. Moreover, structures generally have a thin configuration composed of a metallic material. Consequentially, such structures exhibit both geometrically nonlinear and materially plastic behavior. Thus, an analysis method, capable of analyzing various material states, i.e. elasticity or plasticity, is required.

Co-rotational (CR) formulation is one such approach which is applicable to geometrically nonlinear problems in a structural analysis. It has been established and investigated in a number of studies (Rankin and Brogan, 1989; Felippa and Haugen, 2005; Crisfield, 1996; Hsiao and Yang, 1995; Le et al., 2011; Battini, 2008; Le et al., 2012). Based on the assumptions of a small degree of strain and large displacement, the CR formulation allows an accurate geometrically nonlinear structural analysis. The main advantage of the CR formulation is that it leads to artificial separation between the material nonlinearity and the geometrical nonlinearity. Therefore, a local formulation is required for the small deformational component, and this is done by using the existing finite element hypothesis. This concept was originally developed by Rankin et al. during the derivation procedure of what is known as the element-independent co-rotational (EICR) description (Rankin and Brogan, 1989). Felippa and Haugen (2005) suggested a unified formulation of the CR formulation and discussed its usefulness as related to the EICR concept. In addition, Felippa et al. concluded that the CR formulation would be extremely useful for elements with a simple geometry. Moreover, the artificial separation between the material

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and the geometrical nonlinearity can provide a reasonable solution to the localized failure problem (Felippa and Haugen, 2005).

Regarding the two-dimensional finite element based on the CR formulation, there exist a number of studies regarding planar beam analysis (Hsiao and Yang, 1995; Le et al., 2011). However, the planar beam element is not applicable to structures which are not slender. One alternative may be a planar membrane element. Despite its capability, the CR formulation related to the planar membrane element is somewhat limited. Battini (2008) established the CR formulation for a four-node solid-like quadrilateral element. However, achieving consistent accuracy using the formulation regarding a general four-node element is difficult because the choice of the local element will significantly affect the in-plane bending result. Moreover, when using Battini's formulation, it is not straightforward to extend to existing planar elements including the in-plane rotational degree of freedom (DOF). Planar elements including the in-plane rotational DOF have been suggested in a number of studies. And such successful linear membrane elements, such as the optimal membrane triangles suggested by Felippa (2003), were extended to a geometrically nonlinear analysis by the authors Cho et al. (2016). Accordingly, an additional treatment during the derivation procedure was proposed to take the in-plane rotational DOF into account, while maintaining a unified formulation. Additionally, the derivation procedure based on total Lagrangian description for the relevant time-transient analysis was proposed by the authors Cho et al. (2016). In the dynamic formulation, it was considered to be impossible to derive the inertial terms because of its complex nature (Crisfield, 1990). Thus, many researchers employed the conventional approach such as a constant mass matrix (Crisfield, 1990; Iura and Atluri, 1995) or a lumped mass matrix (Masuda et al., 1987; Xue and Meek, 2001). By extension, Le et al. (2014); 2011) established the consistent dynamic formulation for the two- and three-dimensional beam element. On the other hand, an extension to plastic analysis based on the CR formulation for three-dimensional beam element was suggested by Alsafadie et al. (2011) and Cai et al. (2009); 2010a). Also, there were the several studies related with the shell element (Cai et al., 2010b; Cortivo et al., 2009). The relevant formulation for the planar element has not been seriously considered, although it could be a useful resource during a nonlinear structural analysis. Specifically, for a structure under axi-symmetric boundary condition or undergoing an arbitrary in-plane behavior, the planar elements are more efficient than three-dimensional elements. Moreover, when such a structure is not a slender, the planar membrane elements can be an appropriate tool for the analysis.

The purpose of this paper is to suggest a consistent CR formulation for a time-transient analysis by extending Battini's and Le's approaches (Battini, 2008; Le et al., 2011). To realize this, the formulation for a higher order solid-like planar element (Bathe, 1996) and a triangular/quadrilateral planar element including in-plane rotational DOF (Felippa, 2003; Ibrahimbecovic and Wilson, 1991) is proposed. Subsequently, approaches for the elastoplastic analysis of a planar element are proposed. The formulation of the local coordinate is then modified by introducing a relevant relationship in the localized deformation. As a result, the modified tangent stiffness matrix and internal force vector in a plastic state can be obtained. Moreover, a simplified approach for the elastoplastic analysis of a planar element is proposed by obeying the feature of the CR formulation. In order to predict the plastic state of the planar structure, a plane-stress-projected plasticity model based on the fully implicit predictor/return-mapping algorithm for von Mises criteria is employed (Neto et al., 2008). Finally the present CR formulation is extended to the contact analyses using the Lagrange multiplier (Zienkiewicz et al., 2005). In the validation procedure of the present approaches, four existing local planar elements are employed. The relevant results are investigated with respect to efficiency and accuracy in a general geometrically nonlinear problem and the geometrically nonlinear elastoplastic problem, respectively. During this procedure, a comparison regarding the choice of a local formulation is carefully conducted and demonstrated.

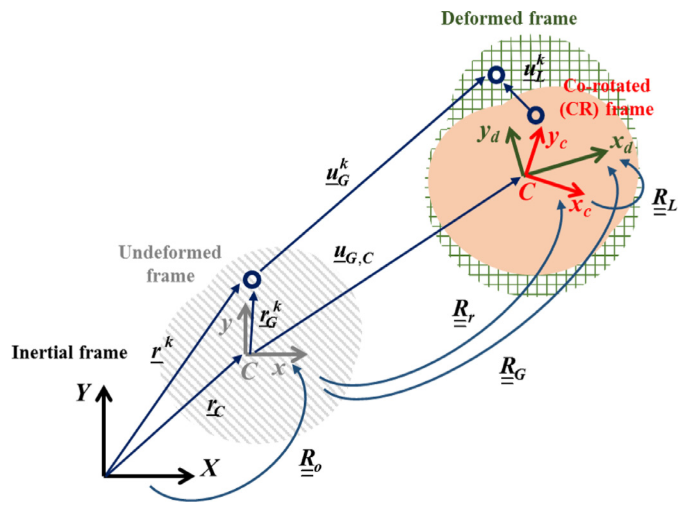

Fig. 1. Coordinates and elemental kinematics in the CR formulation.

2. Co-rotational formulation for a planar element

In this section, a summary pertaining to the CR planar element is given by examining the results of previous studies (Battini, 2008; Cho et al., 2016). The CR formulation can be established by tracking an elemental motion based on the coordinates. The coordinates, located in the centroid of the element, include the inertial, undeformed, the CR, and deformed coordinates. The CR coordinate is an additional intermediate configuration between the undeformed and deformed frame. The local system can be defined between the CR and the local coordinates. Fig. 1 shows the coordinates and related transformations obeying the elemental kinematics regarding an arbitrary point in the planar element. Each transformation can be accomplished by the rotational operators, $\underline{\underline{R}}_G$, $\underline{\underline{R}}_r$ and $\underline{\underline{R}}_L$. The rotational operators, $\underline{\underline{R}}_G$, $\underline{\underline{R}}_r$ and $\underline{\underline{R}}_L$, are composed of the global, rigid-body and local rotations, θ_G, θ_r and θ_L, respectively. Here, the local rotational variables are obtained by multiplying the operators, $\underline{\underline{R}}_L = \underline{\underline{R}}_r^T \underline{\underline{R}}_G \underline{\underline{R}}_o$. Regarding the solid-like planar element, the rotational variables with respect to the local system, i.e., $\underline{\underline{R}}_L$ and θ_L can be ignored.

The rigid body rotation of the element, which consists of the operator, $\underline{\underline{R}}_r$, can be defined by the global nodal translations as follows.

$$\tan\theta_r = \frac{\sum_{k=1}^{N_e} r^k_{G,1} r^k_{d,2} - r^i_{G,2} r^k_{d,1}}{\sum_{k=1}^{N_e} r^k_{G,2} r^k_{d,2} + r^k_{G,1} r^k_{d,1}} \tag{1a}$$

$$\underline{r}^k_d = \left\{ \underline{r}^k + \underline{u}^k_G - \underline{r}_C - \underline{u}_{G,C} \right\} \tag{1b}$$

where, N_e is the number of nodes. By introducing N_e throughout the derivation procedure, it become possible to preserve the nature of the unified CR formulation. By complying with the present kinematics, the elemental matrices and load vectors will be obtained in the foregoing subsections. A complete description can be found in the literature (Battini, 2008; Cho et al., 2016). An additional description regarding the unified form is described in Section 2.2.

The derivations are related to the local and global displacement vectors.

$$\underline{q}_L = \{u^1_L, v^1_L, \theta^1_L, \dots, u^{N_e}_L, v^{N_e}_L, \theta^{N_e}_L\}^T \tag{2}$$

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