ARTICLE IN PRESS
JID: SAS [m5G;May 27, 2017;9:21]

International Journal of Solids and Structures 000 (2017) 1–11

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Approximate estimation of wrinkle wavelength and maximum amplitude using a tension-field solution

Takashi Iwasa

Department of Mechanical and Aerospace Engineering, Tottori University, 4-101, Koyama-cho Minami, Tottori, 680-8551, Japan

ARTICLE INFO

Article history:
Received 26 May 2016
Revised 15 May 2017
Available online xxx

Keywords:
Approximate estimation
Wrinkle wavelength
Wrinkle amplitude
Tension-field theory
Membrane structures

ABSTRACT

Membrane-based space structures such as solar sails, inflatable antennas and sunshields have attracted attention for use on future advanced space science and engineering missions. Since these membrane-based space structures are easily deformed by small disturbances, a practical calculation method for the surface configuration of the deformed membranes is required to realize the membrane structures without failure. This study formulates an equation to estimate the wrinkle wavelength and maximum amplitude appearing on thin membranes based on a tension-field solution. The equation calculates the wrinkle wavelength and maximum amplitude using three wrinkling parameters, which are the major principal stress in the wrinkled region, the length of the wrinkle line, and the wrinkle strain. Because these wrinkling parameters are given by a traditional wrinkling analysis using tension-field theory, the formulated equation reveals an approximate wrinkle wavelength and a maximum amplitude appearing on the thin membranes without cumbersome bifurcation analysis and has characteristics applicable to various types of structures. By conducting a wrinkling analysis using tension-field theory for the two classical membrane models, which have wrinkling phenomena appearing across the entire membrane and in part of the membrane, a wrinkle wavelength and maximum amplitude are estimated from the formula. Comparing the estimated results with those given by wrinkling analysis using shell theory, it is observed that the estimate appropriately captures the wrinkle wavelength and maximum amplitude in the wrinkled region where tension-field theory is applicable. The equation presented in this study offers a new, practical approach to estimate the wrinkled membrane surface configuration.

1. Introduction

A membrane is a flexible and lightweight material whose bending stiffness is considerably lower than its in-plane stiffness. Owing to this distinctive property, such membranes have drawn attention as effective components of ultra-lightweight space structures such as solar sails (Tsuda et al., 2013), inflatable reflectors (Bouzidi and Lecieux, 2012), and sunshields (Clampin, 2008). However, since the membrane only carries an infinitesimal compressive stress, a wrinkling phenomenon immediately occurs as soon as an excessive compressive stress is applied. Once a wrinkling phenomenon appears in a membrane, the operational function of its structures is affected. Thus, prediction of the wrinkling behavior at the design stage is a significant issue to construct the membrane-based space structures reliably. However, since wrinkling phenomena are classified as bifurcations with strong geometrical nonlinearities, a theoretical prediction of nonlinear wrinkling behavior is generally impossible for various types of structures. Accordingly, several computational approaches such as finite-element analysis have been widely applied to reveal the complicated nonlinear wrinkling behavior appearing in various membrane structures. Most presently available computational approaches for wrinkling phenomena are divided into two categories. One wrinkling analysis uses tension-field theory, and the other uses shell theory.

In wrinkling analysis using tension-field theory, a membrane is treated as an ideal material with a negligible bending stiffness that can carry no compressive stress (Wagner, 1931). Owing to this assumption, the wrinkling analysis with tension-field theory approximately calculates stress fields in thin membranes including wrinkles and slacks and clarifies an overall membrane surface configuration using taut, slack, and wrinkled regions. To release the compressive stress appearing in the membranes, several specific analytical approaches including those using a variable Poisson's ratio (Mikulas, 1964; Stein and Hedgepeth, 1961), a modified elasticity matrix (Miller and Hedgepeth, 1982) and a special deformation tensor (Roddeman et al., 1987a), have been developed. These analytical approaches have also been integrated into a finite-element analysis for practical use (Akita and Natori, 2008; Lee and Youn, 2006; Miller et al., 1985; Nakashino and Natori, 2005; Pimprikar

E-mail address: iwasa@mech.tottori-u.ac.jp

http://dx.doi.org/10.1016/j.ijsolstr.2017.05.029

et al., 2010; Roddeman et al., 1987b). Since the finite-element analysis with tension field theory has attracted attention as a practical calculation method for the behavior of a membrane (including wrinkles and slacks) with low computational cost, it has already been applied to the clarification of complicated nonlinear membrane behavior, such as the deformation of airbags (Contri and Schrefler, 1988) and deployment of solar sails (Miyazaki, 2006). However, the wrinkling analysis with tension-field theory has a major disadvantage in that quantitative wrinkle features, such as amplitudes, wavelengths, and number of wrinkles, cannot be revealed. Thus, there is demand for a simplified estimation method for quantitative wrinkle features.

To quantitatively clarify nonlinear wrinkling behavior, a wrinkling analysis using shell theory has been considered during the last two decades. According to this method, a membrane is modeled as a thin-shell material with a small bending stiffness. The appearance of wrinkles is treated as a buckling phenomenon of the membrane, and a quantitative wrinkle feature is clarified from the buckling mode (Lee and Lee, 2006; Miyamura, 2000; Wang et al., 2007). However, since wrinkling analysis using shell theory is quite sensitive to the finite-element discretization of the membranes (Wong and Pellegrino, 2006b), a high computational cost is required to calculate complicated wrinkling behavior. In parallel with shell-theory-based studies, a semi analytical approach has also been developed to estimate the wrinkling behavior of thin membranes (Cerda et al., 2002; Wong and Pellegrino, 2006a). In these studies, Cerda et al. developed a scaling law for wrinkle wavelength and amplitude by solving a minimization problem of the strain energy stored in a stretched, slender elastic sheet. Wong et al. also formulated an equation to calculate wrinkle wavelength and amplitude for two specific membrane models, namely a rectangular membrane subjected to shear loading and a square membrane subjected to a tension-corner loading. These studies make it possible to provide a preliminary quantitative estimate of wrinkling behavior at low computational cost, while the membrane models with the formulated equations are limited.

In this paper, to estimate the wrinkle wavelength and maximum amplitude appearing on various types of membrane structure, a simplified equation using a tension-field solution is formulated based on the analytical model of Wong and Pellegrino. The formula calculates the wrinkle wavelength and amplitude directly from only three wrinkling parameters, namely, the major principal stress in the wrinkled region, the length of the wrinkle line, and the wrinkle strain. Since all of these wrinkling parameters are given by traditional wrinkling analysis using tension-field theory, the formulated equation is applicable to various types of structures, and can approximately estimate a maximum surface distortion of the wrinkled membrane without any difficulty. To compare the effectiveness and applicable range of the proposed equation, the prediction results of the formulated equation were compared with the simulation results of shell theory computed by the finite element analysis.

This paper is structured as follows. In Section 2, an equation for calculating the wrinkle wavelength and amplitude from a tension-field solution is formulated. In Section 3, the two classical membrane models treated in this study, which have wrinkling phenomena appearing across the entire membrane and in part of the membrane, are explained, and an outline of the finite-element analysis using tension-field theory and shell theory is described. In Section 4, wrinkle wavelength and amplitude are estimated from the formula for the two classical membrane models, and their effectiveness is discussed in comparison with the results given by shell-theory-based wrinkling analysis. In Section 5, the major findings obtained in this study are concluded.

2. Formula for wrinkle wavelength and amplitude

In this section, an equation to calculate the wrinkle wavelength and amplitude from a tension-field solution is formulated based on the assumptions of the Wong and Pellegrino analytical model (Wong and Pellegrino, 2006a). This model was proposed as a way to theoretically solve the quantified wrinkling behavior appearing on two specific membrane models, namely a rectangular membrane subjected to shear loading and a square membrane subjected to tension-corner loading. The assumptions used in this model are as follows.

(1) The appearance of a single wrinkle is modeled as a buckling phenomenon of a simply supported, infinitely wide plate;
(2) The minor principal stress in the wrinkled region is equal to the critical buckling stress of the membrane;
(3) Inextensional theory is applied to describe the deformation mechanics of the wrinkles in the directions of the minor principal stresses.

In this study, based on the above three assumptions, an equation for calculating the wrinkle wavelength and amplitude from a tension-field solution is formulated. The new formulation explicitly includes the tension-field solutions. Therefore, it can predict the wrinkle half-wavelengths and amplitudes on various types of membrane models. During this formulation, the first assumption is slightly modified in order to treat partial wrinkles.

2.1. Formula for wrinkle wavelength

Based on the first assumption, the critical buckling stress of the membrane is given by the following equation in Wong and Pellegrino's analytical model.

$$\sigma_{cr} = -\frac{\pi^2 D}{t\lambda^2} \tag{1}$$

Here, t is the membrane thickness, λ is the half-wavelength of the wrinkles and D is the bending stiffness of the membranes, which is

$$D = \frac{E\ t^3}{12\left(1-\upsilon^2\right)} \tag{2}$$

E and υ represent the Young's modulus and the Poisson's ratio of the membranes, respectively. The first assumption is considered to be valid when an elongated single wrinkle is treated, namely that the wavelength of the wrinkles is significantly smaller than the length of the wrinkle line. However, not all of the wrinkles appearing in the membrane are always elongated, and partial wrinkles also appear. In that case, Eq. (1) is not suitable to represent the critical buckling stress for single wrinkles. Thus, in this study, a finite-wide plate model is applied as the appearance model of the single wrinkle.

Fig. 1 illustrates the analytical model used in this study. As shown in the figure, the appearance of a single wrinkle is modeled as a buckling phenomenon of the simply supported finite plate whose edge lengths are λ and l_ξ. Here, l_ξ corresponds to the length of the wrinkle line. In this case, the critical buckling stress of the single wrinkle is written by the following equation.

$$\sigma_{cr} = -\frac{\pi^2 D}{t{l_\xi}^2}\left(\frac{l_\xi}{\lambda}+\frac{\lambda}{l_\xi}\right)^2 \tag{3}$$

Since Eq. (3) is reformulated as follows,

$$\sigma_{cr} = -\frac{\pi^2 D}{t\lambda^2}\left(1+\frac{\lambda^2}{{l_\xi}^2}\right)^2 \tag{4}$$

Please cite this article as: T. Iwasa, Approximate estimation of wrinkle wavelength and maximum amplitude using a tension-field solution, International Journal of Solids and Structures (2017), http://dx.doi.org/10.1016/j.ijsolstr.2017.05.029