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Analysis of the critical velocity of a load moving on a beam supported by a finite depth foundation

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ABSTRACT

A detailed analysis of the critical velocity of a uniformly moving load is carried out in this paper. It is assumed that the load is traversing an infinite beam supported by a finite depth foundation under plane strain condition. The problem is solved analytically in the Fourier domain, respecting two formulations of the interface condition. The critical velocity is determined as the velocity under which the undamped beam deflection tends to infinity, i.e. by identifying double poles in the Fourier image of the beam deflection situated on the real Fourier variable axis. Results obtained are compared with the previously published results of this author, where simplifying assumptions were imposed on the shear contribution of the foundation. Based on the results, an addition to the previous formula for critical velocity estimate is proposed. It is confirmed that there are several critical velocities, but the lowest one, which is the dominating one, smoothly changes from the classical value to the lowest wave-velocity of propagation in the foundation according to the mass ratio defined as the square root of the fraction of the foundation mass to the beam mass. Particular situations, where the first double pole cannot be formed in this model, are discussed in detail. In such cases a significant increase of deflection is observed in the vicinity of the expected location, defining thus a pseudo-critical velocity. Attention is paid to the cases where there are two or more very close critical velocities, which causes an increased risk for practical applications. In addition, several options of damping are compared.

1. Introduction

Response of rails to moving loads is still active field of scientific research and innovation. The simplest model of this kind is formed by a beam supported by an elastic foundation represented by a uniform layer of springs. Two distinct interpretations are used, either the beam is modelled by the rail and the layer of springs represents the underlying remainder of the track structure, or an equivalent beam encompassing the whole track is defined and the spring layer stands for the foundation. In the former case the stiffness of the spring layer along the beam length is named as the track modulus. In both cases this approach defines Winkler's model, which is often referred to as a "one-parameter model".

If the beam is infinite, then the critical velocity is defined as the load velocity which in an undamped situation originates beam infinite displacements. The first solution of steady-state dynamic response of an infinite beam on an elastic foundation traversed by a moving force was presented by Timoshenko (1926). In Frýba (1972) the moving coordinate system is introduced to convert the governing equation to ordinary differential equation that can be solved by the Fourier integral transform. Then the critical velocity is identified as the velocity under which double pole (DP) in the Fourier image of the displacement is formed on the real Fourier variable axis. In Chen and Huang (2000) the concept of the dynamic stiffness matrix is implemented. Two semi-infinite beams are solved for and connected by continuity equations. Then the critical velocity can be determined as the velocity that ensures the nullity of the determinant of the dynamic stiffness matrix. Both approaches naturally give the same result, which for the Euler-Bernoulli beam is written as

$$v_{cr} = \sqrt[4]{\frac{4kEI}{m^2}},\ v_{cr,N} = v_{cr}\sqrt{1-\eta_N},\ \eta_N = \frac{N}{N_{cr}},\ N_{cr} = 2\sqrt{kEI} \quad (1)$$

where v_{cr} is the critical velocity and $v_{cr,N}$ is the critical velocity encompassing the effect of the normal force acting on the beam axis. EI, m and k stand for the bending stiffness and mass per unit length of the beam, and the stiffness of the foundation (Winkler's constant), respectively. η_N is the normal force ratio, N is the normal force (considered positive when compressive) and N_{cr} is the critical value of this force ensuring the existence of the critical velocity, i.e. preventing the beam of instability. The concept described above

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was extended to finite and infinite beams with sudden change in foundation stiffness in Dimitrovová and Rodrigues (2012).

Practical experience showed that the realistic critical velocity can be much lower than the one determined by the classical formula (1), especially on soft soils (Madshus and Kaynia, 2000; Kaynia et al., 2000). This fact originated a significant research with the goal to improve the classical formula (1). Possible generalizations were mainly related to the foundation model. Two different directions have been followed. In one of them, the foundation was replaced by an elastic half-space. In this context, it is necessary to mention the pioneering works by Filippov (1961) and Lansing (1966). The former one is addressing the problem of the critical velocity and the latter one belongs to the first works dealing with a moving load on an elastic half-space. In such a model it has been shown that the critical velocity of the moving load corresponds to the velocity of propagation of Rayleigh waves in the foundation (Krylov et al., 2000). Experimental evidence of such critical velocity is reported in Madshus and Kaynia (2000) and Kaynia et al. (2000). In Dieterman and Metrikine (1997) and Wolfert et al. (1997), it has been concluded that the problem is more complicated, and besides the Rayleigh-wave velocity, there is a critical velocity resulting from the dynamic interaction between the beam and the elastic half-space.

Another direction related to the foundation model generalizations, suggested improvements of the Winkler model (Hetenyi, 1946) by introduction of another parameter in so-called Filonenko–Borodich or Pasternak models (Pasternak, 1954). This parameter is introduced to account for the coupling effect of the Winkler linear elastic springs, it represents the shear contribution and can equally be understood as distributed rotational spring. The model is named as a “two-parameter model”. Further improvements included a finite depth of the foundation. These models are named as the Vlasov and modified Vlasov models (Vlasov et al., 1966; Vallabhan and Das, 1991).

It is important to point out, that only finite active depth of the foundation soils can be included in the analysis. This depth can either be the actual depth at which a stiff substratum (rock) is located or a depth after which no appreciable soil deformations occur. Definition by a rigid base is a common approach in works that implement the Vlasov model (Ozgan, 2013). Determination by detecting the depth where any soil deformations induced by the applied loads are negligible is presented for instance in Mednikov (1965). The dynamically active layer is deduced by fitting the excitation frequency at resonance, determined experimentally, with the lowest natural frequency corresponding to pressure wave caused by the applied pressure. Then this depth is used to define the Winkler and Pasternak parameters. Other works related to railway transportation predicts the active depth according to experimental measurements within the range of 4.5–8 m (Li and Selig, 1995).

Under assumption of finite depth, detailed analyses of induced vibrations by moving loads and of the critical velocities are presented in Metrikine and Vrouwenvelder (2000) for two-dimensional problem and in Metrikine and Popp (2000) in three-dimensions. It was demonstrated that the critical velocity is affected by the interaction between the beam and the foundation and therefore can differ from the velocity of propagation of Rayleigh waves. In agreement, in Náprstek and Fischer (2010) and Dimitrovová (2016) it was highlighted that the interaction between the beam and the foundation can be dominant. In Dimitrovová (2016) only simplified plane models of the foundation were used for analyses of finite and infinite beams, but it was confirmed that the critical velocity is not given either by the classical value from Eq. (1) or by the lowest wave-velocity of propagation in the foundation, but there is a smooth transition between these two values governed by the mass ratio defined as the square root of the fraction of the foundation mass to the beam mass. For a lower mass ratio, the critical velocity approaches the classical formula (1) and for a higher mass ratio (approximately equal to 10), it approaches the lowest wave-velocity of propagation in the foundation. In this paper, generalizations that are consistent with the previous works are derived. Deductions are simplified by considering only a constant moving force, thus the analytical solution can be restricted to its steady-state part. It is shown that in full two-dimensional model, the final result depends on the interface condition between the beam and the foundation. However, differences between the three possibilities: the previously published results in Dimitrovová (2016) and results according to two options for the interface condition are quite similar. Namely, results with the interface condition in form of zero horizontal displacement (ZHD) yield values very similar to the simplified model from Dimitrovová (2016) and results obeying the zero shear stress (ZSS) condition have the asymptotic tendency to slightly lower velocity, approximately the velocity of propagation of Rayleigh waves. In view of these new results, an additional term to the previous formula from Dimitrovová (2016) is proposed, accounting for this complementary wave propagation, caused by introduction of the horizontal displacements in the foundation.

Particular situations, where the first DP cannot be formed in this model, are discussed in detail. In such cases a significant increase of deflection is observed in the vicinity of the expected location, defining thus a pseudo-critical velocity, typical for low shear ratios. Other situations, particularly dangerous in practical applications, are cases with two or more very close critical velocities. These cases are also analysed in detail. In addition, several options of damping are compared.

Thus the new contributions of this paper are:

- Alternative resolution of the problem, which due to a convenient set of dimensionless parameters allows visualization of critical velocities related to practically all possible situations in one graph (readers can find in graphs the value they need, without recalculation of the expressions derived);
- Detailed analysis of the critical velocity under two possible interface conditions: exact determination by identification of DPs;
- New term to be added to the previously published formula for critical velocity estimate covering this extended situation (enhanced formula for the critical velocity, readers can estimate by this formula the value they need, without recalculation of the expressions derived);
- Definition and determination of pseudo-critical velocities (identification of the limitations of the model analysed);
- Identifying the cases with very close critical velocities;
- Analysis of several damping models.

There are apparently no recent works on this subject; two-dimensional analyses under plane strain condition with finite depth foundation and moving loads are given in Theodorakopoulos (2003) where Biot's dynamic poroelastic properties are exploited but the beam is not include in the model; linear elastic solid is implemented by van Dalen et al. (2015), but the focus of the paper is on the abrupt change in foundation stiffness and the beam is omitted in the model. Elastic half-plane is introduced in Ruta (2004). Other papers on three-dimensional analyses with finite depth media and analytical derivations like Besserer and Malischewsky (2004) are focused on different issues. Works directed to realistic simulations of rail transit are usually based on numerical resolution, implementing finite and/or boundary elements (Galvín et al., 2010). Most papers that advance further with analytical resolution are using foundation and elements underneath the rails as discrete springs and dampers, sometimes additional masses are included, but only in form of discrete concentrated elements, and therefore the wave propagation is somehow limited (Sun, 2002;