International Journal of Solids and Structures 000 (2017) 1–10

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

The interfacial analysis of a film bonded to a finite thickness graded substrate

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ARTICLE INFO

Article history:
Received 7 December 2016
Revised 10 April 2017
Available online xxx

Keywords:
Bonded problem
Loading condition
Stress intensity factor
Film/substrate system

ABSTRACT

The problem of an elastic film bonded to a finite-thickness graded substrate under different loading conditions is investigated, in which the shear modulus of the graded substrate is assumed to vary exponentially along its thickness and perfect adhesion is adopted at the contact interface. The governing singular integral equation for the present model is formulated analytically in terms of interfacial shear stress. With the help of the collocation method, the governing equation is further solved numerically. The interfacial shear stress, the normal stress in the film as well as the singularity near the film edges are discussed in order to evaluate the interface behaviors that are closely related to failure and destruction of the film/substrate systems. It is found that the interface behavior of the film/substrate system can be modified by tuning the material and geometric parameters of both the film and the graded substrate. Compared with cases under a non-symmetric loading and a symmetric one, the effect of some parameters is observed to be dependent of the loading type.

1. Introduction

The mechanical behavior of film/substrate systems are of interest in a wide range of industrial, hi-tech and biotechnological applications in recent decades, including, for example, piezoelectric sensors and actuators attached to a structure to monitor and control the deformation and vibration of the host structure (Fang et al., 2013; Gladwell, 1980; Jin and Wang, 2011), stretchable and flexible electronics made of inorganic films and soft substrate (Dai et al., 2015), and etc. A film/substrate system can also be used to uncover the mechanism of cell differentiations, which are affected by the surrounding substrate (Banerjee and Marchetti, 2012). The challenge of establishing suitable models is therefore constantly faced to predict the interface stresses that may result in failure and destruction of the film/substrate systems (Huang et al., 2010).

Various studies have been done to access the interface stress distribution in the film/substrate system. Generally, these works can be categorized into two typical approaches. The first one, represented by Akisanya and Fleck (1994) and Yu et al. (2001), makes an assumption that a pre-existing crack lies on the edge of a thin film, and deals the problem with the fracture mechanics theory. Nevertheless, a lack of pre-existing cracks or defects would be crucial in many cases (Hu, 1979). The second one could be termed as a contact model, in which perfect bonding between the film and the corresponding substrate is assumed and contact mechanics is used to find the stress field at the interface and near the edge of the film. Using a contact model, Arutiunian (1968) studied the contact problem between a half plane and a stiffener with finite length, and gave an infinite power series solution. The same problem was well solved by Erdogan and Gupta (1971) later through tackling the governing singular integral equation into terms of the interface shear stress. Shield and Kim (1992), considering the bending stiffness of the film, employed a beam theory model for a thin film bonded to an elastic half plane to incorporate normal stresses at contact interface to the interfacial shear ones. The multi-layered films or multi-periodic films bonded to an elastic substrate was modeled by Erodgan and Joseph (1990a,b). A closed form solution of the governing singular equation was obtained by Alaca et al. (2002) by adopting Vekua's solution procedure of Prandtl's equation and assuming a nearly rectangular film profile. Jin and Wang (2011) discussed the electromechanical behavior of surface-bonded piezoelectric film attached to an infinite elastic half plane including the adhesive layer. Analysis of stress singularity in thin film bonded structures is considered by Lanzoni (2011) for several geometric configurations under different loading conditions. Recently, the problem of a Timoshenko beam of finite length perfectly bonded to a homogeneous isotropic half plane loaded by

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http://dx.doi.org/10.1016/j.ijsolstr.2017.04.029
0020-7683/

Please cite this article as: P. Chen et al., The interfacial analysis of a film bonded to a finite thickness graded substrate, International Journal of Solids and Structures (2017), http://dx.doi.org/10.1016/j.ijsolstr.2017.04.029

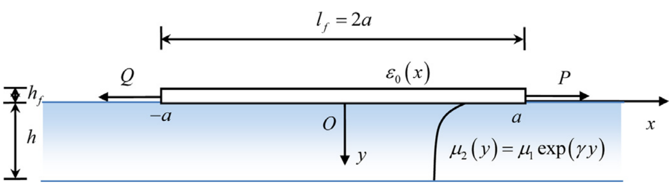

Fig. 1. The two-dimensional non-slipping contact model between an elastic film of length l_f and a finite-thickness graded substrate. h is the length of the graded substrate.

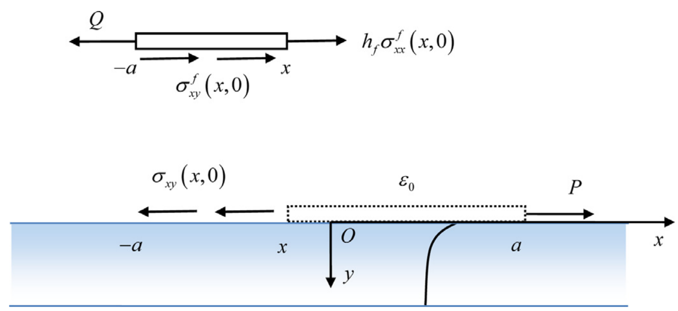

Fig. 2. Schematic of the mechanical behavior of the bonded interface between a film and an elastically graded substrate.

concentrated forces and couples is considered by Lanzoni and Radi (2016).

All the bonded models mentioned above involve homogeneous materials. Materials with gradient variation along certain direction have been widely found in nature, for example, teeth and bones (Suresh, 2001). What is more, functionally graded materials (FGM) have attracted numerous attentions of scientists due to their novel performance (Giannakopoulos and Pallot, 2000; Qian et al., 2009) in various present and potential applications. The traditional contact problem between a stamp or punch and a graded medium have been explored extensively to find the contact stress that would cause crack initiation of wear of surface. These works include Booker et al. (1985), Guler and Erdogan (2004, 2007), Choi and Paulino (2008, 2010), Ke and his co-workers (2006, 2008), El-loumi et al. (2010), Chen and Chen (2013a,b), Dag et al. (2012), Chen et al. (2015), Jin et al. (2013), etc. In the mentioned works, contact stresses at the contact region are mainly focused. As for the sub-surface stresses and local deflection of a graded medium loaded by a pre-determined pressure or a rigid punch, Chidlow et al. (2011, 2012) proposed a valid analytical method.

Although both experimental studies and theoretical researches on the contact mechanics of graded materials have been reported, existing literature is mainly concerned with the traditional indentation problem, and few works have discussed the bonded problem between a deformable layer and a graded medium. Guler (2008) and Guler et al. (2012) explored the contact problem between a thin film and a graded/FGM coated half plane using both FEM and analytical methods. Recently, Chen et al., (2016a, b) investigated the contact problem of an elastic film subjected to a mismatch strain on a finite-thickness graded substrate. It was found that the interfacial behavior is significantly influenced by the thickness of the graded substrate. Whether the result under a mismatch strain loading condition is consistent with that under a non-symmetric load when a film bonded to a finite-thickness graded substrate? How can we tune different material and geometric parameters to improve the interfacial behavior?

In order to answer the above questions, a non-slipping contact model is established in this paper, in which an elastic film attach to a graded substrate of finite thickness under a non-symmetric loading condition. The governing integro-differential equation for the present model is formulated analytically in terms of interfacial shear stress, and is further solved numerically with the help of the collocation method. The interfacial shear stress, the normal stress in the film as well as the singularity near the film edges are mainly discussed in order to evaluate the interface behavior that is closely related to failure and destruction of the film/substrate systems

2. The bonded model of a thin film on a finite-thickness graded substrate

The two-dimensional non-slipping contact model between an elastic film and a finite-thickness graded substrate which is fixed on a rigid foundation is shown in Fig. 1, where the length of the film is l_f=2a. h_f, μ_f, ν_f are denoted as the thickness, the shear modulus and the Poisson's ratio of the film, respectively, and h is the thickness of the graded substrate.

The shear modulus of the graded substrate is assumed to abide by

$$\mu_2(y) = \mu_1 \exp(\gamma y),\ \ 0 \le y \le h, \tag{1}$$

where μ_1 is the value of shear modulus at the surface of the graded substrate. γ is a constant characterizing the inhomogeneity of material, which can be expressed as

$$\gamma = \frac{1}{h} \ln\left(\frac{\mu_3}{\mu_1}\right), \tag{2}$$

where μ_3 corresponds to the shear modulus at the bottom of the graded substrate. The exponential function is commonly used to describe a graded medium in existing theoretical models, and it covers a fairly broad class of graded materials, for example, graded γ-TiAl/Y-TZP and Ni-Al_2O_3 (Suresh, 2001; Suresh et al., 1997). When $\gamma = 0$ is chosen, the finite-thickness graded substrate will reduced to a homogeneous elastic one. Note that the problem of a thin-film bonded to an elastic layer subjected to a thermal variation has been well solved by Lanzoni and Radi (2009). In the present model, the Poisson's ratio of the graded substrate is assumed to be a constant ν due to the neglectable effect.

3. Governing equation of the present model

For the present plane contact problem, th equilibrium equations of the graded substrate in absence of body forces can be written as (Chen et al., 2016b),

$$(\kappa+1)\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 v}{\partial x \partial y} + (\kappa-1)\frac{\partial^2 u}{\partial y^2} + \gamma(\kappa-1)\frac{\partial u}{\partial y} + \gamma(\kappa-1)\frac{\partial v}{\partial x} = 0, \tag{3}$$

$$(\kappa-1)\frac{\partial^2 v}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + (\kappa+1)\frac{\partial^2 v}{\partial y^2} + \gamma(3-\kappa)\frac{\partial u}{\partial x} + \gamma(\kappa+1)\frac{\partial v}{\partial y} = 0, \tag{4}$$

where $u(x, y)$ and $v(x, y)$ are the displacement components in $x$and y directions, respectively, $\kappa = 3 - 4\nu$ is adopted for the plane strain case and $\kappa = (3 - \nu)/(1 + \nu)$ for the plane stress one.

Taking the Fourier transform of Eqs. (3) and (4), and carrying out a lengthy mathematic analysis similar to Chen and Chen (2013b), the surface displacements of the graded substrate $u(x, 0)$ and $v(x, 0)$ can be expressed as

$$\frac{\partial u(x,0)}{\partial x} = -\frac{\kappa+1}{4\pi\mu_1}\int_{-a}^{a}\frac{\sigma_{xy}(r,0)}{r-x}\mathrm{d}r + \frac{\kappa-1}{4\mu_1}\sigma_{yy}(x,0) + \frac{1}{\pi}\int_{-a}^{a}[K_{11}(x,r)\sigma_{xy}(r,0) + K_{12}(x,r)\sigma_{yy}(r,0)]\mathrm{d}r. \tag{5}$$

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