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Mixed mode fracture analysis using generalized averaged strain energy density criterion for linear elastic materials

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ABSTRACT

A modified energy-based model called the generalized averaged strain energy density (ASED) criterion is proposed to predict the mixed mode I/II fracture resistance of brittle and quasi-brittle materials. The proposed criterion takes into account the effect of *T*-stress in addition to the stress intensity factors K_I and K_{II} when calculating the strain energy density averaged in a control volume around the crack tip. The effect of *T*-stress on the mixed mode fracture resistance of materials is investigated by using the generalized ASED criterion and the generalized SED criterion. The results predicted by these criteria are compared with the experimental results reported in the literature from fracture tests conducted on the Brazilian disc specimens and the semi-circular bend specimens made of Harsin marble and Guiting limestone. It is shown that the theoretical estimates of the generalized ASED criterion are consistent very well with the experimental results. The theoretical results also show that the specimens having a negative *T*-stress require higher loads in order their averaged strain energy density around the crack tip attains its critical value.

1. Introduction

Numerous failure criteria have been proposed by many researchers to assess the onset of brittle fracture in linear elastic materials under mixed mode loading. These criteria usually consider stresses or energy as key parameters for predicting the fracture load in brittle and quasi-brittle materials such as ceramics, rocks, glasses and brittle polymers. Erdogan and Sih (1963) proposed a stress-based model called the maximum tangential stress (MTS) criterion which was widely employed by researchers. The maximum energy release rate (G) and the minimum strain energy density (SED) criteria suggested by Hussain et al., (1974) and Sih (1974), respectively, are also two well-known energy-based fracture criteria. These three criteria consider only the effect of singular stress terms of William's series expansion in order to predict the kinking angle and the onset of brittle fracture under mixed mode loading.

In several research studies, a significant discrepancies have been reported between the theoretical predictions of the conventional mixed mode fracture criteria and the experimental results. Some researchers suggested that these discrepancies can be related to ignoring the effects of higher order terms of the William's series expansion around the crack tip when stress-based fracture theories like MTS are utilized. The first higher order term is the *T*-stress, a constant term parallel to the crack plane, which is independent of the distance from the crack tip. Williams and Ewing (1972) were the first to employ the MTS criterion to evaluate the effect of *T*-stress on mixed mode brittle fracture but only in a plate with an angled crack, where closed form solutions were available for the mode I and mode II stress intensity factors (K_I and K_{II}) and the *T*-stress.

Later, a generalized maximum tangential stress (GMTS) criterion based on three crack parameters K_I, K_{II} and T, was proposed by Smith et al., (2001) to evaluate the effect of *T*-stress on brittle fracture under general mixed mode loading conditions. They showed that the *T*-stress has a significant influence on both the fracture initiation angle and the onset of fracture. A large number of experiments conducted by Ayatollahi and his coworkers on different brittle or quasi-brittle materials revealed that the generalized MTS criterion is able to provide very good estimates for the fracture initiation angle and the onset of mixed mode fracture in comparison with the conventional MTS criterion (Aliha and Ayatollahi, 2011, 2012; Ayatollahi and Aliha, 2006, 2009). More recently a number of energy-based fracture criteria were proposed based

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Nomenclature

$A(r)$	in-plane area of control volume
a	crack length
B	biaxiality ratio
dW/dA	strain energy density function
E	axial modulus of elasticity
$E(r)$	elastic deformation energy
$\bar{E}$	averaged elastic deformation energy on the area $A(r)$
E_{cr}	critical mean value of elastic deformation energy
G	shear modulus of elasticity
K_I	mode-I stress intensity factor
K_{Ic}	mode-I fracture toughness
K_{If}^{I}	mode-I fracture resistance under mode I loading
K_{If}	critical mode-I stress intensity factor corresponding to the fracture load
K_{II}	mode-II stress intensity factor
K_{IIf}	critical mode-II stress intensity factor corresponding to the fracture load
K_{eff}	effective stress intensity factor
M_e	mixity parameter
L	half span in SCB specimen
P	applied load
R	radius of SCB and BD specimens
r, θ	polar coordinates with the origin located at the crack tip
r_c	radius of control volume
S	strain energy density factor
T	T-stress
T_f	critical T-stress corresponding to the fracture load
t	thickness of SCB and BD specimens
T^*	dimensionless form of T-stress
T_I^*	dimensionless form of T-stress under pure mode I loading
Y_I	mode I geometry factor
Y_{II}	mode II geometry factor
Y_I^I	mode I geometry factor under pure mode I loading
κ	elastic constant introducing the stress state in the model
$\sigma_{ij}(i,j=r, \theta, z)$	stress components
σ_t	tensile strength
ν	Poisson's ratio
α	dimensionless form of the radius of control volume
β	crack inclination angle with respect to loading direction
β_{II}	crack inclination angle corresponding to pure mode II loading
ASED	averaged strain energy density criterion
BD	Brazilian disk specimen
GASED	generalized averaged strain energy density criterion
GMTS	generalized maximum tangential stress criterion
GSED	generalized strain energy density criterion
MTS	maximum tangential stress criterion
SCB	semi-circular bend specimen
SIF	stress intensity factor

on the leading terms of the William's series expansion governed by the stress intensity factors (SIFs) K_I and K_{II} together with the T-stress (Ayatollahi et al., 2015a, 2015b). For instance, Ayatollahi et al. (2015a) suggested a modified mixed mode fracture model called the generalized strain energy density (GSED) criterion and successfully estimated a large number of experimental data. The strain-based criterion has been applied to various brittle materials to study their fracture mechanisms (Wu, 1974; Nalla et al., 2003). The maximum tangential strain (MTSN) criterion proposed by Chang (1981) and has been utilized to study mixed mode fracture in angled center crack problem. The MTSN criterion states that crack propagates in the direction where the tangential strain reaches its maximum value at a critical distance from the crack tip. A modified form of the traditional MTSN criterion based on three crack parameters K_I, K_{II} and T, was later proposed by Ayatollahi and Abbasi (2001).

For brittle fracture in mode I, Chao and Zhang (1997) used two fracture mechanics parameters, K_I and A_3 (a second non-vanishing term of the William's series expansion) to provide a better explanation for the onset of fracture in brittle materials. Some other researchers showed that there is a direct influence of T-stress on brittle fracture when the size of the plastic zone is very small (Williams and Ewing, 1972; Ueda et al., 1983). Cotterell (1966) and Cotterell and Rice (1980) reported that the second and third terms of Williams' series control the stability of crack direction and crack propagation. Leguillon and Murer (2008a) revisited the Cotterell and Rice theory (1980) on kinking of a crack subjected to biaxial loading in a homogeneous material. They suggested that even under a symmetric loading, the T-stress at a crack tip can induce crack kinking. The T-stress has also been shown to have an influence on the crack kinking out of a biomaterial interface (Leguillon and Murer 2008b). Using modified energy-based fracture models, Ayatollahi et al. recently investigated the effect of T-stress on the fracture trajectory and the fracture resistance of brittle and quasi-brittle materials under mode I loading (Ayatollahi et al., 2015b, 2016).

Lazzarin and his co-authors proposed another energy-based fracture model called the averaged strain energy density (ASED) criterion which utilizes the deformation energy averaged in a control volume of radius r_c to estimate the fracture load for both cracked and notched specimens (Lazzarin and Zambardi, 2001). In crack problems, the control volume has a circular shape and its radius is a material property. According to the ASED criterion, brittle fracture takes place when the averaged value of the deformation energy inside the control volume reaches a critical value (which again is a material property). Lazzarin et al. applied the ASED criterion to different notched specimens (e.g. U- and V-shape notches) and to various materials in order to predict the fracture load under mixed mode loading (Berto and Lazzarin, 2007, 2014; Berto et al., 2012). They proposed a mathematical expression to calculate the radius of control volume r_c by using the mode I fracture toughness K_{Ic}, the ultimate tensile strength σ_t and the Poisson's ratio ν of the material (Lazzarin and Zambardi, 2001).

Lazzarin and his co-researchers employed finite element analysis to obtain the average value of strain energy density around the crack tip. Another way to obtain the strain energy density is application of the equations of stresses/strain based on the well-known Williams' series expansion. The classic theories of fracture mechanics only considers the singular terms of Williams' expansion and these conventional criteria underestimated or overestimated the fracture strength of the experimental results.

In the present paper, a generalized form of ASED criterion (GASED) is proposed to estimate the mixed mode fracture resistance of brittle and quasi-brittle materials. The criterion takes into account both the singular terms and the first non-singular term related to the stresses and strains in the William's series expansion.