International Journal of Solids and Structures 120 (2017) 257–267

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Hyperbolic hardening model for quasibrittle materials

Pantelis Liolios*, George Exadaktylos

School of Mineral Resources Engineering, Technical University of Crete, University Campus, GR-73100, Chania, Greece

ARTICLE INFO

Article history:
Received 16 June 2016
Revised 27 April 2017
Available online 8 May 2017

Keywords:
Yield condition
Constitutive law
Elastic plastic solid
Cohesive frictional material
Geomechanics

ABSTRACT

In this study an elastoplastic hardening model for quasibrittle materials is presented. The yield surface exhibits hyperbolic meridians while its shape on the deviatoric plane is described by an elliptic function. The proposed hardening mechanism is controlled by the slope of the asymptotes of the hyperbolic meridians and thus by the mobilized friction of the material. The yield surface is capped during the first stages of hardening and opens up before reaching the peak strength. On the deviatoric plane the hardening is non-uniform producing changes in both the size and the shape of the yield surface. The proposed plastic potential is related with the yield function through a simple modification of its volumetric part. The model is equipped with a hardening rule that is a monotonically increasing elliptic function of the hardening parameter. The latter is given in its rate form and it is pressure dependent. A proposed ductility rule controls this pressure dependency and leads to brittle behavior in tension and ductile behavior at high confinement compression. All the parameters have physical interpretation and are directly related with measurable mechanical properties of the material in the lab. Most of them could be identified from a uniaxial compression test. The predictions of the model are compared against experimental datasets and it is shown that exhibit very good agreement.

1. Introduction

Quasibrittle materials, like rocks and concrete are granular materials with cohesion and internal friction. In the tensile regime their mechanical response is nearly perfectly brittle. On the contrary, in the compression regime they exhibit certain ductility which increases with the mean pressure. During low confinement compression the material initially hardens and then softens. Moreover, a degradation of the elastic moduli is observed mainly in the softening regime. Additionally, the material exhibits some amount of plastic dilatation due to the propagation and coalescence of cracks. At high compression stresses, only compaction phenomena are observed and the material exhibits remarkable ductility with continuous hardening (Jirásek and Bažant, 2002). Thus, the elastoplastic response of the quasibrittle materials is quite complicated and difficult to be modeled. To capture these phenomena two major approaches have been developed: models that incorporate only the elastoplasticity theory and models that combine the elastoplasticity theory with damage mechanics. The first approach models both hardening and softening by controlling the evolution of the yield surface through appropriate hardening/softening rules. The second approach usually models the hardening in the frame of elastoplasticity and considers perfect plasticity post-peak. The softening response is modeled in the frame of damage mechanics theory. Nevertheless, both approaches are quite similar in modeling the pre-peak behavior. Concentrating the attention on the latter, key role in the development of hardening models plays the selection of the yield surface, the hardening rule and the plastic potential.

Several yield surfaces have been proposed to model the mechanical behavior of quasibrittle materials. One of the most used yield surface models is the Mohr–Coulomb (Coulomb, 1776; Mohr, 1900). Mohr–Coulomb is a simple two parameter model that predicts a linear relationship between the normal and the shear stress during yielding/failure. The shape of the Mohr–Coulomb yield surface in the principal stress space is an irregular cone with three-fold symmetry. The meridians of the yield surface are linear while its trace on the deviatoric plane is an irregular symmetrical hexagon. Finally, its two parameters are directly related with the cohesion and internal friction of the material. The Mohr-Coulomb is simple and thus widely-used model; however, it has four main disadvantages: (a) it overestimates the tensile strength, (b) the yield surface is not smooth, (c) it does not consider the influence of the intermediate principal stress which has a significant influence on the peak strength of the material (e.g. Mills and Zimmerman, 1970; Mogi, 1971; Takahashi and Koide, 1989) and (d) it is an open surface, thus hydrostatic or nearly hydrostatic compres-

* Corresponding author.
E-mail addresses: pantelis@mred.tuc.gr (P. Liolios), exadakty@mred.tuc.gr (G. Exadaktylos).

http://dx.doi.org/10.1016/j.ijsolstr.2017.05.011

sion loadings cannot be modeled (Chen and Han, 1988; Jirásek and Bažant, 2002). To overcome the first disadvantage, Hoek and Brown (1980) and Hoek et al. (2002) proposed a two parameter model for rocks with parabolic meridians which is a variant of the original Leon (1935) model developed for concrete. Willam and Warnke (1974) proposed two variants of a yield surface model, namely, one with three parameters which has linear meridians and one with five parameters which has parabolic meridians. The trace of both models on the deviatoric plane is described by an elliptic function and they both account for the influence of the intermediate principal stress. Pramono and Willam (1989) introduced a capped variant of the Leon model for concrete and later Etse and Willam (1994) combined the Pramono–Willam model with the elliptic function of the Willam–Warnke model to produce a smooth yield surface which degenerates to the Menétrey and Willam (1995) failure surface. This yield surface has been exploited by many authors who successfully modeled quasibrittle materials (e.g. Grassl and Jirásek, 2006; Grassl et al., 2013; Kang, 1997; Kang and Willam, 1999; Unteregger et al., 2015). Starting from the soil mechanics discipline, Kim and Lade (1984) and later Ewy (1999) proposed a variation of the Lade–Duncan model (Lade and Duncan, 1975) in order to include the cohesion. The three parameter Kim–Lade model has curvilinear meridians while the simpler two parameter model by Ewy has linear meridians. Other worth mentioning proposed phenomenological yield surfaces are the capped three parameter Bresler–Pister model (Bresler and Pister, 1958) which is an extension of the Drucker–Prager model (Drucker and Prager, 1952), the capped seven parameter Bigoni–Piccolroaz model (Bigoni and Piccolroaz, 2004) and the van Eekelen model (van Eekelen, 1980; Parisio et al., 2015) among others.

The second important feature in modeling the mechanical behavior of quasibrittle materials during hardening is the evolution of the yield surface. Rocks and concrete may exhibit elastoplastic behavior that depends on the stress path, as well as on the value of the mean pressure and the Lode angle. It has been observed that the accumulation of plastic deformation leads to a more brittle response at the low confinement regime. On the contrary, at high confinement pressures the material remains ductile (Bažant et al., 1999; Caner and Bažant, 2000). To model such a complicated behavior an appropriate pressure dependent hardening rule is required that controls the evolution of the yield surface. Some models targeting in a specific pressure regime frequently discard the pressure dependent ductility (Sulem et al., 1999; Lubliner et al., 1989; Lee and Fenves, 1998; Voyiadjis et al., 2008). Other models incorporate only the pressure dependence (Etse and Willam, 1994; Pramono and Willam, 1989; Kang and Willam, 1999) or only the Lode angle dependence (Lin et al., 1987). The more complicated models include the effect of both the confining pressure and the Lode angle (Grassl and Jirásek, 2006; Grassl et al., 2013; Unteregger et al., 2015; Parisio et al., 2015; Paliwal et al., 2017).

Finally, a hardening model must be supplemented by an appropriate plastic potential for the flow rule. It is known that in the case of quasibrittle materials the associated flow rule over-predicts the plastic volumetric strains (Chen and Han, 1988; Jirásek and Bažant, 2002). Therefore, a non-associated flow rule is preferable. There are several choices that can be adopted for the plastic potential function. However, in many cases a plastic potential that is derived as a volumetric modification of the yield function has been proposed (e.g. Pramono and Willam, 1989; Etse and Willam, 1994; Grassl and Jirásek, 2006; Paliwal et al., 2017; Mas and Chemenda, 2015).

In this work an elastoplastic hardening model for quasibrittle materials is presented, i.e. the model is restricted to the up to peak elastoplastic behavior. The yield surface is based on the hyperbolic failure criterion recently proposed by Liolios and Exadaktylos (2013b); 2013a). The meridians of this yield surface in the principal stress space are hyperbolas, while its shape on the deviatoric plane is described by the elliptic function of the Willam–Warnke model. The surface is smooth and accounts for the influence of the intermediate principal stress, i.e. the strengthening of the material due to the support of the intermediate principal stress. The proposed hardening mechanism is controlled by the slope of the asymptotes of the hyperbolic meridians and thus by the mobilized friction of the material. The yield surface is initially capped and eventually it opens up at the side of hydrostatic compression before the peak. Hence, it may be used for the modeling of hydrostatic or nearly hydrostatic compressive loadings. The expansion of the yield surface on the deviatoric plane is non uniform that is it changes both its size and its shape. The model predicts pressure dependent ductility which leads to brittle behavior in the tensile regime and ductile behavior for high confinement stress fields. Furthermore, the model is supplemented with a plastic potential that is based on the yield surface. By assuming deviatoric normality, the volumetric parts of the plastic potential and the yield function are related through a simple expression. All the parameters of the model have physical interpretation and are directly related with measurable mechanical properties of the material in the lab. A single uniaxial compression test is required to calibrate almost all the parameters apart from two. These last two parameters require the calibration of the failure surface on a set of experiments or an ad-hoc estimation. Finally, the predictions of the model are compared against experimental datasets and exhibit very good agreement.

2. Conventions and definitions

In this study the sign convention used in engineering mechanics will be followed, that is the compression stresses will be considered as negative and tensile stresses as positive quantities, respectively. Consequently, the strains that tend to compact the material will be considered to be negative. Moreover, it will be assumed that

$$\sigma_1 \geq \sigma_2 \geq \sigma_3 \tag{1}$$

where $\sigma_i \quad (i = 1, 2, 3)$ represent the principal stresses.

The yield surface function will be presented as an expression of the octahedral stresses and the Lode angle. It may be recalled that the octahedral normal stress p, the octahedral shear stress T and the Lode angle θ are given by the following expressions

$$p = \frac{1}{3} I_1 \tag{2}$$

$$T = \sqrt{\frac{2}{3} J_2} \tag{3}$$

$$\cos 3\theta = \frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}} \tag{4}$$

where

$$I_1 = \sigma_{kk} = \sigma_1 + \sigma_2 + \sigma_3 \tag{5}$$

is the first invariant of the stress tensor. J_2 and J_3 are the second and third invariants of the stress deviator tensor s_{ij}, respectively

$$s_{ij} = \sigma_{ij} - p\delta_{ij} \tag{6}$$

$$J_2 = \frac{1}{2} s_{ij} s_{ji} \tag{7}$$

$$J_3 = \det s_{ij} = \frac{1}{3} s_{ij} s_{jk} s_{ki} \tag{8}$$

with $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$.