International Journal of Solids and Structures 000 (2017) 1–16

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Numerical study on sub-harmonic generation due to interior and surface breaking cracks with contact boundary conditions using time-domain boundary element method

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ARTICLE INFO

Article history:
Received 9 October 2016
Revised 24 April 2017
Available online xxx

Keywords:
Nonlinear ultrasonic testing
Contact acoustic nonlinearity
Sub-harmonic wave
Numerical simulation
Time-domain boundary element method

ABSTRACT

Recently, the ultrasonic testing (UT) based on the contact acoustic nonlinearity (CAN) has attracted notice as a new technique for detection of closed cracks which cannot be detected by using the linear UT (LUT). In the nonlinear UT (NLUT), detection and size measurement of flaws are conducted via the frequency spectrum analysis of nonlinear ultrasonic waves which consist of higher- and sub-harmonic waves. Although the mechanism of higher-harmonic generation due to CAN has been understood mostly from theoretical and experimental points of view, that of sub-harmonic generation has not been revealed yet. In addition, there are few numerical and theoretical studies on the phenomenon of sub-harmonic generation due to the CAN. In this paper, the boundary integral equation for two-dimensional (2-D) elastic wave scattering by cracks is formulated and numerically solved to investigate the behavior of the sub-harmonic waves. The clapping motion and dynamic friction on crack faces are modeled by considering contact boundary conditions, and their interaction with the characteristic of frequency response in the corresponding linear system is investigated. In this study, time-domain numerical simulations are implemented for both an interior crack in an unbounded elastic solid and a surface breaking crack in an elastic half-space. Some frequency response analyses for a linear system corresponding to the time-domain nonlinear simulations are also performed by using the frequency-domain boundary element method. From obtained numerical results, the causes of sub-harmonic generation are discussed.

1. Introduction

Periodic nondestructive evaluation (NDE) of in-service structures such as nuclear plants, bridges, and tunnels is important to detect flaws at an early stage and to prevent accidents. Among many techniques of NDE, ultrasonic testing (UT) and radiographic testing (RT) are often used for the flaw detection inside materials. UT has some advantages in terms of portability and safety compared with RT. Therefore, UT is considered as more appropriate for the inspection of infrastructures.

However, it is difficult to detect closed cracks at an early stage by using the conventional linear UT (LUT) that is based on the acoustic impedance mismatch between materials and flaws. This is because an incident wave is hardly scattered on contacting crack faces, and adequate scattered waves cannot be received in the measurement of LUT. On the other hand, a nonlinear UT (NLUT) based on the contact acoustic nonlinearity (CAN) has recently attracted notice as a new technique for detection of closed cracks (Solodov et al., 2011). This method is based on nonlinear ultrasonic waves which consist of higher- and sub-harmonics of the fundamental frequency. The nonlinear ultrasonic waves are considered as generated by the CAN, and they are sensitive to degradation of material properties at the early stage of damage.

There are two different mechanisms of higher-harmonic generation, and they have been mostly understood from theoretical and experimental points of view (Solodov et al., 2011). The first mechanism is described by a non-symmetrical stress-strain diagram under perpendicular vibration to a discontinuous interface such as a closed crack. The stiffness crossing the discontinuous interface for compression is harder than that for tension. The clapping motion between crack faces is caused under the situation, and it excites higher-harmonic waves of both odd and even orders. The

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http://dx.doi.org/10.1016/j.ijsolstr.2017.07.029
0020-7683/

other one is derived from energy dissipation due to the interface roughness. The friction force is generated under some compressive stress, and then an incident wave is distorted in a symmetrical way. It is known that odd order higher-harmonic waves are generated by this dynamic friction phenomenon on crack faces.

In contrast to higher-harmonic waves, the generation mechanism of sub-harmonic waves has not been understood clearly from a theoretical point of view yet. The sub-harmonic generation is much more complex physical phenomenon than the higher-harmonic one. The higher-harmonic waves can be generated not only by the interaction between the incident wave and the flaw but also by the interaction between the transducer and the test material. However, it is empirically considered that the sub-harmonic waves are generated only by the former interaction. Therefore, more accurate detection and size measurement of flaws are expected if the sub-harmonic waves can be used in the NLUT. In order to investigate the generation mechanism of sub-harmonic waves and to control this phenomenon, further theoretical and/or numerical approaches are demanded.

The generation of sub-harmonic waves due to CAN was first observed by Solodov and Vu (1993). At that time, it seemed that the sub-harmonic wave could be generated just before the chaotic vibration and an unstable phenomenon. However, comparatively stable sub-harmonic waves have been observed by Yamanaka et al. (2004) in the ultrasonic measurement for a surface breaking crack, and the depth measurement technique has been developed by Ohara et al. (2011). Through experiment and one-dimensional (1-D) numerical simulation, moreover, Hayashi and Biwa (2013) have shown that sub-harmonic waves are generated due to a thin layer between two solid blocks. However, the generation mechanism of sub-harmonic waves has not been understood clearly in all cases as previously stated. It is difficult to intuitively comprehend the generation mechanism of sub-harmonic wave unlike that of higher-harmonic wave.

As previous theoretical and numerical approaches on the NLUT based on CAN, the 1-D analysis of clapping motion on the discontinuous interface has been first implemented by Richardson (1979). Thereafter, some higher-harmonic simulations have been implemented for the modeling of the clapping motion and dynamic friction on the crack faces. For instance, 2-D numerical simulations have been conducted by a time-domain boundary element method (BEM) (Mendelsohn and Doong, 1989; Hirose, 1994). They have used Coulomb's friction law with constant static and dynamic friction coefficients. Also, the 3-D axisymmetric problem, i.e. a penny-shaped crack subjected to normal incidence of P wave, has been solved numerically (Hirose and Achenbach, 1993). On the other hand, the 2-D antiplane simulation using the finite element method (FEM) for the discontinuous interface with dynamic friction conditions has been implemented by Meziane et al. (2011). The 2-D numerical simulation for cracks only with clapping motion has also been conducted by Kimoto and Ichikawa (2015) using the finite difference method (FDM). In addition, some 3-D sub-harmonic simulations have been conducted for delamination of thin layer using the FDM (Sarens et al., 2010) and FEM (Delrue and Abeele, 2012). However, they have presented the sub-harmonic generation only in the case that the thickness of delaminated layer is much smaller than its diameter and specifically less than one-twentieth.

From the previous studies, it seems that understanding the characteristic of *frequency response in the corresponding linear system* is important for revealing the sub-harmonic generation mechanism due to CAN. Therefore, we concentrate upon the resonance characteristics of the system containing cracks in the numerical simulation. We consider the cases of an interior crack in an unbounded elastic solid and a surface breaking crack in an elastic half-space. The effect of interaction between the frequency response characteristic and contacting crack faces on the sub-harmonic generation is investigated.

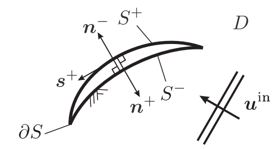

Fig. 1. Elastic wave scattering by a crack in infinite domain *D*.

Taking into account the numerical approaches for the NLUT, accurate calculation of the physical quantities on the crack faces such as displacement and stress is required because they significantly affect the states of crack faces such as contact and non-contact. Moreover, appropriate treatment of boundary conditions on crack faces is also important for high precision solutions. Therefore, a BEM is desirable because it can exactly deal with the nonlinear boundary conditions on crack faces, and accuracy of the BEM for physical quantities on boundaries is much better than that of FEM and FDM. In addition, a BEM has attractive advantage to deal with infinite and semi-infinite domains. To numerically solve the time-dependent problems with nonlinear boundary conditions, a time-domain BEM is utilized in this study. For accurate and stable BEM calculation, we use the convolution quadrature method (CQM) based on the implicit Runge–Kutta scheme (IRK-based CQM) (Lubich and Ostermann, 1993) and the Galerkin method for discretization with respect to time and space, respectively. In abbreviation, we name the IRK-based convolution quadrature time-domain BEM as an IRK-based CQ-BEM.

The rest of this paper is organized as follows: First, the problem statement and IRK-based CQ-BEM formulation are presented. Second, the nonlinear boundary conditions on crack faces are described. Then, the numerical procedure implemented in this research is explained. Finally, some numerical results are shown and discussed.

2. Problem statement and boundary integral formulation

We consider two situations in this study. The first situation is wave scattering by a crack in an unbounded elastic solid, and the other is the one by a surface breaking crack in an elastic half-space. In both cases, the base material containing cracks is assumed to be a 2-D, homogeneous, isotropic, and linearly elastic solid. Therefore, only CAN on boundaries is considered as nonlinearity in the presented simulations, and the superposition principle of field variables, displacement, stress, and strain, is available in a domain. In this study, the far-field approximation of scattered wave is introduced to evaluate the received wave because the scattered wave is often measured in far-field in NDE.

Since the Galerkin method and IRK-based CQM (Lubich and Ostermann, 1993) are used for discretization with respect to space and time, respectively, the variational boundary integral equations (BIEs) with respect to space are presented here.

2.1. Wave scattering by an interior crack

Let S be a smooth curved crack surface in $\mathbb{R}^2$ as depicted in Fig. 1. S consists of S^+ and S^- called positive and negative sides of crack face, respectively ($S = S^+ \cup S^-$ and $S^+ \cap S^- = \emptyset$). $\boldsymbol{n}^+$ and $\boldsymbol{n}^-$ are the unit normal vectors to S^+ and S^-, respectively. $\boldsymbol{s}^+$ is the unit tangential vector to S^+. Disregarding the body force, we consider the problem to find the displacement solution $\boldsymbol{u}$ in $D(\equiv \mathbb{R}^2 \backslash S)$ which satisfies the following equations:

$$c_T^2 \nabla^2 \boldsymbol{u}(\boldsymbol{x}, t) + \left(c_L^2 - c_T^2\right) \nabla\nabla \cdot \boldsymbol{u}(\boldsymbol{x}, t) = \ddot{\boldsymbol{u}}(\boldsymbol{x}, t) \qquad \boldsymbol{x} \in D, \tag{1}$$