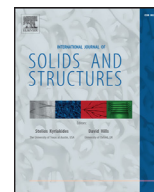




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Dynamic end effects in an orthotropic strip

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ABSTRACT

Spatial extent of dynamic end effects in an orthotropic semi-infinite strip is investigated within the framework of linear elastodynamic theory. Formulation of the dynamic response of a strip was utilized to examine the effect of frequency of excitation and constitutive properties of the strip on the upper bound on a region affected by end effects. Analysis included examination of materials spanning a wide range of material anisotropy. It was found that frequency has only marginal effect on the extent of dynamic end effects in materials with high degree of orthotropy up to the first cut off frequency. Extremely small extent of end effects was found in strips made of orthotropic material with fibers oriented perpendicular to longitudinal direction of a strip.

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1. Introduction

End effects in beam-like and plate-like members are source for much concern to engineers of structures. That concern is especially troubling in structures made of composites and laminates (e.g., Horgan, 1982; Miller and Horgan, 1995a; Horgan, 1996a). The study of end effects is traditionally separated into static and dynamic end effects, commonly treated separately. Studies of the static end effects in fibrous composites, associated with Saint Venant's principle (e.g., Horgan, 1972; Choi and Horgan, 1977; Horgan and Simmonds, 1994), disclose high non-locality of these effects and are considered to be well understood (e.g., Miller and Horgan, 1995b; Horgan, 1996b and references therein). Dynamic end effects, on the other hand, deserved only sparse attention even within the classical isotropic domain (Karp and Durban, 2011, 2013). Among these, only few examine end effects in composites.

The extent of dynamic end effects in anisotropic cylinders was investigated by Huang and Dong, (1984) and in shells by Bhattacharayya and Vendhan, (1991). Dynamic edge effects in laminated composite plates were studied by Dong and Huang, (1985). These few studies provide an important contribution to the analogy between the static and dynamic end effects. The possible effect of end effects on natural frequency of vibration of a beam was investigated by Duva and Symmonds, (1991). Study of that type does not refer explicitly to the spatial extent of end effects. Experimental study of dynamic end effects in orthotropic material in a context of structural health monitoring was performed by Gecht, (2014). The results obtained revealed an increased penetra-

tion depth of dynamic end effects measurable using simple equipment.

All these studies suggest that dynamic end effects in composites have general similarity to the static end effects in these materials. Yet, the complete understanding of the effect of frequency on the extent of end effect in conjunction with the effect of degree of orthotropy is still demanding.

The purpose of the present study is to disclose the explicit effect of the degree of orthotropy and frequency of the excitation on the extent of dynamic end effects. That study is related to the question of validity of Dynamic Saint Venant's Principle (DSVP) in structures and might have implication to structural health monitoring (SHM). That aim is accomplished by considering the dynamic steady state response of a strip made of orthotropic material subjected to an unspecified harmonic end excitation while the strip is held in plane conditions. Elastic behavior of the material is assumed to be free of any viscous damping. Solution of that problem was obtained analytically within the framework of linear elastodynamics.

Exact upper bound of the spatial decay distance (lowest decay rate) of dynamic end effects has been found for strips made of several structural orthotropic composites representing wide span of orthotropy levels. The dependence of that upper bound on frequency of excitation has been mapped exposing the underlying pattern of the combined material-frequency effect. Unexpectedly, spatial extent of end effects smaller than in an isotropic material by 30 percent has been found to characterize several materials. That low penetration depth is found to occur in highly orthotropic materials with fibers oriented 90° to the strip axis.

Section 2 outlines formulation of the problem of an elastic waveguide with free lateral surfaces. That formulation leads to

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frequency equation, solution of which, for several orthotropic materials, is given in Section 3. Discussion of the results is suggested in Section 4, followed by concluding remarks in Section 5.

2. Formulation of the problem

Consider a semi-infinite strip with a thickness $2h$ made of homogeneous, orthotropic, elastic material that occupies the region $x \geq 0$, $|y| \leq h$. Assume that the planes of material symmetry coincide with the strip axes, either being the fibers collinear with x axis making $E_{xx} = E_{LL}$ or perpendicular to it with $E_{xx} = E_{TT}$. The strip can be held in plane strain ($|z| \rightarrow \infty$) or plane stress ($|z| \ll h$) conditions with z coordinate not active while the faces $y = \pm h$, $x \geq 0$ are free of traction. We seek the dynamic response of the strip subjected to an unspecified harmonic excitation at the end $x = 0$ with frequency ω as a parameter.

Formulation of that problem of a semi-infinite strip under plane conditions made of orthotropic material, leading to frequency equations, can be found in several monographs and articles (e.g., Buchwald, 1959; Auld, 1973; Solie and Auld, 1973; Rose, 1999). That derivation is recapitulated here briefly adopting notation used by Karp and Durban, (2005).

2.1. Dynamic response of a semi-infinite strip

Dynamic response of a strip is governed by equation of motion

$$\nabla \cdot \boldsymbol{\sigma} = \rho \ddot{\mathbf{u}} \quad (1)$$

where $\boldsymbol{\sigma}$ is Cauchy stress tensor, ρ is the homogenized mass density, ∇ is the gradient vector and \mathbf{u} is the displacement vector which has two components

$$\mathbf{u} = u\mathbf{i} + v\mathbf{j} \quad (2)$$

for the two dimensional problem posed, where both components u , v depend only on x and y coordinates and time, t . Here \mathbf{i} and \mathbf{j} are the unit vectors in the x and y directions, respectively.

For a plane problem, the linear constitutive relations are written in the form

$$\begin{aligned} \sigma_x &= S_{11}\varepsilon_x + S_{12}\varepsilon_y \\ \sigma_y &= S_{21}\varepsilon_x + S_{22}\varepsilon_y \\ \tau_{xy} &= S_{66}\gamma_{xy} \end{aligned} \quad (3)$$

with strain components given by

$$\varepsilon_x = \frac{\partial u}{\partial x}; \quad \varepsilon_y = \frac{\partial v}{\partial y}; \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}. \quad (4)$$

Substitution of (3) and (4) into (1) leads to equation of motion written in terms of displacements alone

$$\begin{aligned} S_{11} \frac{\partial^2 u}{\partial x^2} + S_{66} \frac{\partial^2 u}{\partial y^2} + (S_{12} + S_{66}) \frac{\partial^2 v}{\partial x \partial y} &= \rho \frac{\partial^2 u}{\partial t^2} \\ (S_{12} + S_{66}) \frac{\partial^2 u}{\partial x \partial y} + S_{66} \frac{\partial^2 v}{\partial x^2} + S_{22} \frac{\partial^2 v}{\partial y^2} &= \rho \frac{\partial^2 v}{\partial t^2} \end{aligned} \quad (5)$$

General solution of these two equations is given in the form of separation of variables

$$\mathbf{u}(x, y, t) = \mathbf{U}(y) e^{i(\xi x - \omega t)} \quad (6)$$

with transversal form of the wave

$$\mathbf{U}(y) = U_x(y)\mathbf{i} + U_y(y)\mathbf{j}$$

Here ξ is the wave number, ω is circular frequency, and $\mathbf{U}(y)$ is the associated cross-sectional profile (wave mode) for both velocity components. Substitution of (6) into (5) leads to a system of two

ordinary differential equations for $\mathbf{U}(y)$

$$\begin{aligned} -(\xi^2 S_{11} - \rho \omega^2) U_x + S_{66} U''_x + (S_{12} + S_{66}) i \xi U'_y &= 0 \\ (S_{12} + S_{66}) i \xi U'_x - (\xi^2 S_{66} - \rho \omega^2) U_y + S_{22} U''_y &= 0 \end{aligned} \quad (7)$$

where prime (') denotes differentiation with respect to transversal coordinate y .

The cross-sectional profile functions $\mathbf{U}(y)$ are determined from the combined requirement to satisfy Eq. (7) and boundary conditions of free surfaces

$$\sigma_y = \tau_{xy} = 0 \quad (8)$$

at $y = \pm h$. That combination of requirements leads to frequency equation relating the wave number ξ to frequency ω . For the purpose of generality, these dimensional parameters will be replaced in the sequel by their non-dimensional counterparts defined by

$$k \equiv \frac{2h}{\pi} \xi, \quad \Omega \equiv \frac{2h}{\pi C_T} \omega. \quad (9)$$

where k and Ω are the non-dimensional wave number and frequency, respectively and C_T is the shear phase velocity defined by $C_T^2 = \frac{S_{66}}{\rho}$.

2.2. Frequency equation

Assuming general solution for system of Eq. (7) in the form

$$U_x(y) = A e^{i\Gamma \xi y}, \quad U_y(y) = B e^{i\Gamma \xi y} \quad (10)$$

leads to characteristic equation

$$S_{22} S_{66} \Gamma^4 - d \Gamma^2 + (S_{11} - S_{66} C^2) S_{66} (1 - C^2) = 0 \quad (11)$$

where

$$d \equiv S_{11} S_{22} + S_{66}^2 - (S_{12} + S_{66})^2 - S_{66} (S_{22} + S_{66}) C^2 \quad (12)$$

with non-dimensional phase velocity C defined as

$$C \equiv \frac{\Omega}{k} \quad (13)$$

The two roots (Γ_1 , Γ_2) of the characteristic Eq. (11) are given explicitly by

$$\Gamma_{1,2} = \sqrt{\frac{d \pm \sqrt{d^2 - 4 S_{22} S_{66}^2 (S_{11} - S_{66} C^2) (1 - C^2)}}{2 S_{22} S_{66}}}. \quad (14)$$

Due to symmetry of boundary data (8), and in view of (14), solution (10) can be separated into symmetric and antisymmetric fields, with symmetric part written in the form

$$U_x = A_1 \eta_1 \cosh\left(\Gamma_1 \frac{\pi k y}{2h}\right) + A_2 \eta_2 \cosh\left(\Gamma_2 \frac{\pi k y}{2h}\right) \quad (15a)$$

$$U_y = A_1 \sinh\left(\Gamma_1 \frac{\pi k y}{2h}\right) + A_2 \sinh\left(\Gamma_2 \frac{\pi k y}{2h}\right) \quad (15b)$$

Here (A_1 , A_2) are integration constants, and

$$\eta_p = \frac{i(S_{22} \Gamma_p^2 - S_{66} (1 - C^2))}{\Gamma_p (S_{12} + S_{66})} \quad p = 1, 2 \quad (16)$$

The wave forms (15) should comply with boundary conditions on the long surfaces (8).

Applying free boundary conditions (8) leads to the transcendental equation

$$\tanh\left(\Gamma_1 \frac{\pi k}{2}\right) - \left(\frac{Q_1}{Q_2}\right)^{\pm 1} \tanh\left(\Gamma_2 \frac{\pi k}{2}\right) = 0 \quad (17)$$

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