International Journal of Solids and Structures 110–111 (2017) 67–79

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A model for creep of porous crystals with cubic symmetry

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ARTICLE INFO

Article history:
Received 25 July 2016
Revised 16 December 2016
Available online 5 February 2017

Keywords:
Porous material
Creep
Single crystal
Anisotropy
Lode parameter

ABSTRACT

A model for description of the creep response of porous cubic single crystal is presented. The plastic potential is obtained by specializing the orthotropic potential of Stewart and Cazacu (Int. J. Solids Struct., 48, 357, 2011) to cubic symmetry. The crystal matrix material response is characterized by power law creep. The predictions of this porous plastic constitutive relation are presented for various values of stress triaxiality (mean normal stress divided by Mises effective stress) and various values of the Lode parameter L (a measure of the influence of the third invariant of the stress deviator). A strong influence of crystal orientation on the evolution of the creep strain and the porosity is predicted. For loadings along the $< 100 >$ directions of the cubic crystal, void growth is not influenced by the value of the Lode parameter. However, for loadings such that the maximum principal stress is aligned with the [110] direction there is a strong influence of the values of the Lode parameter and the fastest rate of void growth occurs for shear loadings (one of the principal values of the applied stress deviator is zero). For loadings such that the maximum applied stress is along the [111] crystal direction the fastest rate of void growth corresponds to L=-1, while the slowest rate corresponds to L=1. These predictions are compared with corresponding predictions of the three dimensional finite deformation unit cell analysis of Srivastava and Needleman (Mech. Mater., 90, 10, 2015). It is found that the phenomenological model predicts the same trends as the cell model calculations and, in some cases, gives good quantitative agreement.

1. Introduction

As is well-appreciated, porosity nucleation, growth and coalescence is the main mechanism of ductile fracture in structural metals. Porosity evolution can also play a role in determining the deformation response of structural metals and alloys in circumstances where fracture is not a main concern but the overall deformation is, for example in deformation processing of sintered materials. Therefore, a basic understanding of the evolution of porosity and its effect on the overall mechanical response is of widespread interest. In a wide range of circumstances, the voids of interest are of a size (say several microns and larger) where the surrounding material can be appropriately characterized by a continuum plasticity constitutive description.

There is a long history of continuum based unit cell model calculations of void containing solids as well as of the development of phenomenological theories of porous plastic solids, see Tvergaard (1990), Needleman et al. (1992) and Benzerga and Leblond (2010) for reviews with a focus on ductile fracture applications. Most of the constitutive relations that have been developed for porous plastic solids have presumed that the surrounding matrix material can be regarded as isotropic. There are circumstances, however, where anisotropy of the matrix material may play a significant role, for example, for strongly textured polycrystalline solids and where the void is embedded in a single crystal or is surrounded by a few crystals. Phenomenological anisotropic porous plastic flow potentials have been developed, for example, by Ahzi and Schoenfeld (1998); Benzerga and Besson (2001); Stewart and Cazacu (2011); Morin et al. (2014); Paux et al. (2015).

Using rigorous limit-analysis theorems, Benzerga and Besson (2001) derived an analytic yield function for a porous material containing randomly distributed spherical voids in a matrix obeying Hill (1948) orthotropic criterion. Within the same framework, Monchiet et al. (2008) studied the case of ellipsoidal voids and derived a closed-form orthotropic plastic potential. The combined effects of anisotropy and tension-compression asymmetry induced by twinning or non-Schmid effects on the dilatational response of porous textured polycrystals was investigated by Stewart and Cazacu (2011), Cazacu and Stewart (2013). These authors analytically solved a limit-analysis problem for the cases of spherical and cylindrical void geometries, respectively, and developed appropriate orthotropic plastic potentials. However, for the case of porous

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http://dx.doi.org/10.1016/j.ijsolstr.2017.02.002
0020-7683/

single crystals analytical derivation of a plastic potential poses challenges that, at least to now, have been insurmountable. If the plastic deformation of the fully-dense single crystal is described using either a Bishop and Hill (1951) type model or the regularized form proposed by Arminjon (1991) with the exponent $n \neq 2$, the plastic dissipation cannot be expressed in closed-form. Therefore, it is impossible to solve the limit-analysis problem analytically and derive a closed-form expression for the plastic potential. This was stated in Paux et al. (2015), who proposed an ad-hoc modification of the Gurson (1975) isotropic yield function.

An alternative approach to model the mechanical response of porous materials is based on the homogenization method developed for non-linear composites by Ponte-Castaneda (see for example, Ponte Castaneda (2002)). This method is based on the equivalence response of the solid under consideration with a linear-comparison composite solid described by a potential quadratic in stresses. It was applied by Idiart and Castaeda (2007) for the study of porous single crystals containing cylindrical voids subject to anti-plane loadings, and more recently by Mbiakop et al. (2015) for two-dimensional plane strain loadings. As pointed out by Mbiakop et al. (2015), for anisotropic crystal plasticity based on a power-type law description with exponent $n \neq 2$, no analytic solution exists even for hydrostatic loadings. Nevertheless, Mbiakop et al. (2015) were successful in obtaining numerical plastic potential surfaces for various loadings. However, no results were reported for the time evolution of plastic strain or porosity under creep loading. For the case of a porous single-crystal with a matrix obeying a quadratic (i.e. $n = 2$) Bishop and Hill (1951) relation, Han et al. (2013) used the linear-comparison composite solid method to obtain an approximate analytical plastic potential that is quadratic in the components of stress. Han et al. (2013) also compared the model predictions with finite-element cell calculations for different crystal orientations. Three-dimensional cell model calculations exploring the effect of crystal induced anisotropy on the stress state dependence of porosity evolution were reported in Wan et al. (2005); Yu et al. (2010); Ha and Kim (2010); Yerra et al. (2010); Lebensohn and Cazacu (2012); Han et al. (2013); Srivastava and Needleman (2012, 2013, 2015).

Finite element modeling of the plastic deformation of single crystals for example fcc crystals requires accounting for slip on each of the twelve available slip systems. This additional computational complexity limits the use of such a model in applications. In addition, although for a rate independent single crystal obeying a Schmid slip system relation the yield surface (and hence the flow potential) is faceted and has sharp corners, rate dependence rounds off the corners and gives rise to a smoother flow potential surface when multiple slip systems are significantly active, Rice (1970).

The aim of this paper is to provide a simple phenomenological model for representing the creep response of porous cubic single-crystals. In particular, for simplicity and to keep the expressions close to those derived analytically from limit analysis, we account for crystal anisotropy but not for the discreteness of slip systems. The proposed phenomenological model is obtained by specializing the orthotropic potential derived by Stewart and Cazacu (2011) to the case of cubic symmetry. To account for rate-effects, we use the approach proposed by Pan et al. (1983). We compare the predictions of the proposed phenomenological model with the three dimensional single crystal unit cell results of Srivastava and Needleman (2015). Their finite deformation finite element calculations were carried out for an fcc single crystal containing a single initially spherical void. The deformation of the matrix was modeled by a crystal plasticity (Asaro and Needleman, 1985) framework with a power law viscous creep relation for the matrix material. The unit cell was subject to creep loading, i.e. a fixed stress state, for a range of values of the imposed stress triaxiality, the ratio of the first to second stress invariants, and a range of imposed values of the Lode parameter, a measure of the third stress invariant.

The results of Srivastava and Needleman (2015) showed a strong effect of anisotropy and stress state on the evolution of the overall creep strain and porosity. As expected, the predicted response was found to be sensitive to the value of the applied stress triaxiality. For the [100] crystal orientation that gives rise to nearly isotropic response, no effect of the Lode parameter on the dilatational response was observed. On the other hand, for anisotropic crystal orientations, a significant influence of the Lode parameter was found on the creep response of the porous crystals even at a high value of the stress triaxiality.

In this paper, using the proposed phenomenological model the effect of crystal orientation is analyzed for various creep loading conditions. The model predictions for the overall creep response and for porosity evolution are compared with the corresponding cell model results of Srivastava and Needleman (2015). Our analytical results show a strong effect of crystal orientation and imposed stress state on the evolution of overall creep strain and of porosity. The two important distinction between the cell model calculations and the simple phenomenological model are: (i) in the cell model calculations of Srivastava and Needleman (2015) the orientations of the slip systems evolve, whereas in the results based on the plastic potential of Stewart and Cazacu (2011) the anisotropy is fixed throughout the deformation history; (ii) the cell model calculations account for void-void interactions, whereas in the simple phenomenological model any such interactions are ignored. Nevertheless, key features of the phenomenological predictions are consistent with those obtained from the cell model calculations.

2. Formulation

To describe the creep response of porous cubic single-crystals, we specialize the orthotropic plastic flow potential of Stewart and Cazacu (2011) to cubic symmetry. The plastic potential of Stewart and Cazacu (2011) is briefly described in Section 2.1 and a model for creep of porous crystals with cubic symmetry is proposed in Section 2.2.

2.1. The plastic potential for orthotropic porous solids of Stewart and Cazacu (2011)

Stewart and Cazacu (2011) used a kinematic limit analysis approach in conjunction with the Hill-Mandel lemma (Hill, 1967; Mandel, 1972) to derive an analytical expression for the plastic potential of an orthotropic rate independent plastic solid containing randomly distributed spherical voids. The plastic behavior of the matrix (void-free solid) was taken to be governed by a relation that accounts for plastic tension-compression asymmetry but is pressure-insensitive, (Cazacu et al. (2006)).

The Stewart and Cazacu (2011) plastic potential has the form

$$\phi\left(\sigma_{ij}, f\right) = \hat{m}^2 \sum_{i=1}^{3} \left(\frac{|\tilde{\sigma}_i| - k\tilde{\sigma}_i}{\sigma_x^T} \right)^2 + 2f\cosh\left(\frac{3\sigma_m}{h\sigma_x^T} \right) - (1 + f^2) \quad (1)$$

where f is the void volume fraction (or porosity), k is a material parameter accounting for the tension-compression asymmetry in plastic deformation, σ_x^T is the uniaxial tensile yield strength along an axis of orthotropy, $\sigma_m = \mathrm{tr}\left(\sigma_{ij}\right)/3$ and σ_{ij} are Cartesian components of the Cauchy stress tensor. In Eq. (1), $\tilde{\sigma}_1$, $\tilde{\sigma}_2$, $\tilde{\sigma}_3$ are the principal values of the transformed stress tensor

$$\tilde{\sigma}_{ij} = \kappa_{ijkl}\sigma'_{kl} \quad (2)$$