International Journal of Solids and Structures 110–111 (2017) 80–88

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Effect of graded interphase on the coefficient of thermal expansion for composites with spherical inclusions

Roberta Sburlati[a,*], Maria Kashtalyan[b], Roberto Cianci[c]

[a] Department of Civil, Chemical and Environmental Engineering (DICCA), University of Genova, Via Montallegro 1, Genova 16145, Italy
[b] Centre for Micro- and Nanomechanics (CEMINACS), School of Engineering, University of Aberdeen, Scotland, UK
[c] DIME - Sez. Metodi e Modelli Matematici, University of Genova, Piazzale Kennedy, pad.D, Genova 16129, Italy

ARTICLE INFO

Article history:
Received 27 September 2016
Revised 19 December 2016
Available online 3 February 2017

Keywords:
Elasticity
Particle reinforced composites
Thermal properties
Spherical inclusions

ABSTRACT

The paper is concerned with the quantitative characterisation of the effective coefficient of thermal expansion for a particulate composite containing spherical inclusions surrounded by an interphase zone, whose properties are graded in the radial direction. A thermo-elastic problem of uniform heating is studied for a single hollow spherical inclusion embedded in a finite matrix assuming power-law variation of the thermo-elastic properties. An exact solution of the problem is derived using hypergeometric functions. The effective coefficient of thermal expansion is determined in closed form for composites with graded interphase zone around hollow and solid inclusions, as well as for the case of void in a graded matrix. Numerical results highlighting the effect of the interphase properties on the coefficient of thermal expansion for different volume fractions of inclusions are presented and discussed.

1. Introduction

Properties of particulate composite materials are greatly affected by the degree of adhesion at the interface between inclusions and the matrix, as is often evidenced by experimental observations. The mismatch of properties between the composite's phases may lead to creation of microcracks, impurities, local porosities and stress concentrations around inclusions.

To describe the effect of these phenomena on composite's properties, a number of different micromechanical models have been proposed in the literature, with some authors introducing an interphase zone between inclusions and the matrix, with properties that differ from those of both main phases.

For spherical inclusions and all phases being homogeneous isotropic, experimental results showed, in particular for polymeric materials and concrete, that properties of the interphase zone are not uniform but vary radially outward from the centre of the inclusion (i.e. Holliday and Robinson, 1973; Sideridis and Papanicolaou, 1988; Lutz et al., 1997). A number of researchers adopted this model and tried to predict the mechanical properties of particulate composites, assuming a specific profile for the properties of the interphase zone and employing various methods.

In 1991, Hashin proposed a three-phase sphere model (Hashin, 1991) which assumes a thin interphase zone surrounding each inclusion, with uniform elastic properties that are different from the properties of both matrix and inclusion. He proposed replacement technique to calculate the properties depending on the volume fraction of inclusions. Several authors have extended this method to the graded interphase zone; this was done by subdividing the interphase zone into a number of concentric homogeneous layers, each having different properties, and using them to simulate a specific profile of the graded interphase (n-layered sphere model) (Herve, 2002; Lombardo, 2005; Gusev, 2014).

A different methodology was proposed by Shen and Li (2003); 2005). The authors adopted an effective interphase model (EIM) and a uniform replacement model (URM) to study the effect of an homogeneous interphase with elastic properties that vary in radial direction. The basic idea of this approach is to replace the inclusion with its surrounding multi-layered interphase with a homogeneous inclusion and to increase the thickness of the interphase in an incremental, differential manner with homogenisation at each step. Sevostianov and Kachanov (2007) and Sevostianov (2007), utilised the Shen and Li's methodology to study the effect of interphase layers on the overall elastic and conductive properties of matrix composite and introduced modifications to the previous methodology to better describe composites with nanoinclusions.

In 1996, Lutz and Zimmerman proposed to model the interphase zone around an inclusion as a matrix material with properties that vary in the radial direction according to the

* Corresponding author.
E-mail addresses: roberta.sburlati@unige.it (R. Sburlati), m.kashtalyan@abdn.ac.uk (M. Kashtalyan), roberto.cianci@unige.it (R. Cianci).

http://dx.doi.org/10.1016/j.ijsolstr.2017.02.001
0020-7683/

power law and asymptotically approach the value of the homogeneous matrix at infinity (Lutz and Zimmerman, 1996). Later, Zimmerman and Lutz (1999), studied thermo-mechanical problems in an uniformly-heated functionally-graded cylinder. They were the first to show that thermo-mechanical-problems involving radially inhomogeneous materials could be solved using hypergeometric functions.

Adopting Lutz and Zimmerman's interphase model, Sburlati and Cianci (2015) determined the bulk modulus of particulate composites with solid and hollow spherical inclusions. A closed-form solution was obtained using hypergeometric functions, and an explicit expression for the effective bulk modulus was obtained following Christensen (2005). A detailed parametric investigation of this expression for non-dilute inclusions was presented in Sburlati and Monetto (2016).

In the present paper, we study the effect of size and properties of the interphase zone on the coefficient of thermal expansion (CTE). CTE plays a critical role in design of composite materials for extreme thermal environments (Sevostianov, 2011). The paper approaches the problem of determining CTE along the lines developed in the previous work by the authors (Sburlati and Cianci, 2015). However, in order to explore the influence of the size of the interphase zone, we consider the graded interphase zone of finite size, with thermo-elastic properties that vary according to the power law and match properties of the matrix at its interface with the matrix, while remaining distinct from the properties of the inclusion at its interface with the inclusion. In the last aspect, the graded interphase model used in the present paper differs from the graded interlayers investigated by the authors previously (Kashtalyan et al., 2009; Kashtalyan and Menshykova, 2009; Sburlati et al., 2014; Sburlati and Kashtalyan, 2016), the properties of which exactly match those of adjacent layers.

To this end, a single composite sphere of a particulate composite with hollow spherical inclusions is considered. The exact analytical solution to the thermo-elasticity problem of uniform heating is derived using hypergeometric function theory (Erdelyi et al., 1953). The exact CTE expressions are obtained for solid and hollow inclusions in terms of the interphase zone properties and inclusion volume fraction following Christensen (2005).

The obtained solution can be also used to describe voids with imperfect interfaces in porous materials (Hatta et al., 2000), two-phase composites with graded inclusions (Lombardo, 2005) and thin-walled hollow inclusions with damages in their walls, i.e. syntactic foams (Tagliavia et al., 2010).

When the properties at the interface with the inclusion are taken equal to the properties of the matrix, the exact expression for the CTE obtained in the present paper approaches the CTE expression obtained by Levin (1967) for a composite with two homogeneous phases.

Numerical results for solid inclusions, hollow inclusions and voids, at a range of phase volume fractions, are presented, and the effects of size and properties of the interphase zone on the CTE are established and discussed.

It is important to emphasise that recent interest to such models is triggered not only by desire to predict the effective properties of particulate composites more accurately, but also by the drive to improve performance of nanocomposites, in which thin inhomogeneous coatings around nanoparticles can be used to increase thermo-mechanical properties of the nanocomposites. It was shown that for nanocomposites thin coatings at the interfaces between constituents of a composite material can make a substantial difference to their functional characteristics and reliability (Sevostianov and Kachanov, 2006; Zappalorto et al., 2011; Anisimova et al., 2016).

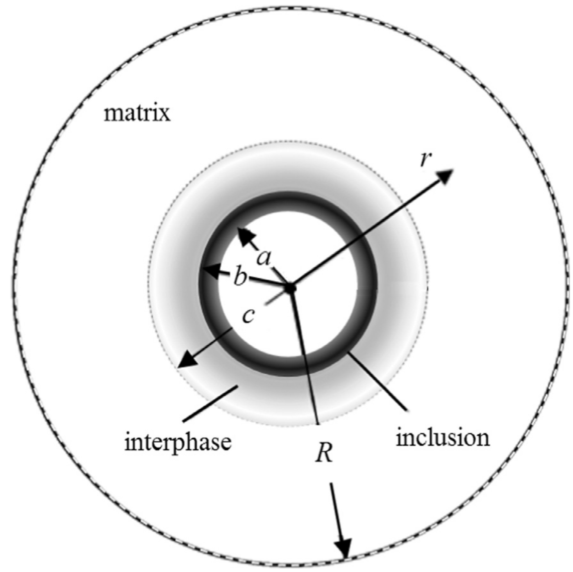

Fig. 1. Micro-mechanical model of the composite sphere.

2. Problem formulation

Fig. 1 shows a single hollow composite sphere embedded in a finite matrix, in according to decribe the composite spheres model for a nondilute elastic suspension of spherical particles (Hashin, 1962; Christensen, 2005; Nemat-Nasser and Hori, 1993). Let the outer radius of the composite sphere be R, and the inner and outer radii of the inclusion be a and b. In this way, a solid inclusion and a void can be viewed as particular cases of a hollow inclusion, when $a = 0$ and $a = b$ respectively. The solution for a void in a matrix may be also adopted to describe a two-phase composite sphere with a functionally graded hollow spherical inclusion.

The inclusion and the matrix are assumed to be homogeneous isotropic, and referred to a spherical co-ordinate system (0; r, θ, ϕ). The elastic properties of the inclusion are denoted as λ_i, μ_i, while the coefficient of thermal expansion (CTE) as α_i. Corresponding properties of the matrix are denoted λ_m, μ_m and α_m.

In order to investigate the interphase zone between the inclusion and the matrix, let us introduce a layer of radius c around the spherical inclusion. We assume that the elastic and thermal properties of this layer vary in the radial direction, exactly matching properties of the matrix, in which the inclusion is embedded, at the interface $r = c$. At the same time, the interface $r = b$ between the inclusion and the interphase zone remains distinct and clearly defined. The elastic and thermal properties of the interphase zone at $r = b$ are denoted as λ_{ip}, μ_{ip} and α_{ip}. The elastic and thermal properties of the interphase zone at its interface with the matrix are the same as those of the matrix.

Assuming power-law variation in the radial direction, the elastic and thermal properties of the graded interphase zone are described as follows

$$\lambda(r) = \lambda_1 + \lambda_2 \left(\frac{b}{r}\right)^{\beta}, \quad \mu(r) = \mu_1 + \mu_2 \left(\frac{b}{r}\right)^{\beta},$$
$$\alpha(r) = \alpha_1 + \alpha_2 \left(\frac{b}{r}\right)^{\beta}, \tag{2.1}$$