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Dynamic fracture analysis by explicit solid dynamics and implicit crack propagation

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ABSTRACT

Combining time-dependent structural loading with dynamic crack propagation is a problem that has been under consideration since the early days of fracture mechanics. Here we consider a method to deal with this issue, which combines a set-valued opening-rate-dependent cohesive law, a quasi-explicit solver and the eXtended Finite Element Method of representing a crack. The approach allows a propagating crack to be mesh-independent while also being dynamically informed through a quasi-explicit solver. Several well established experiments on glass (Homolite-100) and Polymethyl methacrylate (PMMA) are successfully modelled and compared against existing analytical solutions and other approaches in 2D up until the experimentally observed branching speeds. The comparison highlights the robustness of ensuring energy is conserved globally by treating a propagating phenomenological crack-tip implicitly, while taking advantage of the computational efficiency of treating the global dynamics explicitly.

1. Introduction

The kinetics of crack propagation is of considerable importance in a large variety of areas from predicting crack arrest length in engineering structures, earthquakes and bone fracture, to impact fragmentation protection in spacecraft and military armour. The majority of modelling approaches to date have assumed that the material response is independent of crack propagation (Crump et al., 2017; Freund and Hutchinson, 1992; Meyers, 1994). This is equivalent to considering structures as time-independent continua subject to instantly applied changes in boundary conditions. In reality, materials behave differently at different length scales. This length-scale dependency ultimately leads to an element of discreetness at some material specific scale. The result is a delay in displacement propagation from application of boundary loading, to an incident point of interest, such as a crack tip. This is where the field of dynamic fracture mechanics aims to bridge the gap between material (continuum) dynamics and crack propagation (an extension of a discontinuity) by considering dynamically-loaded cracks, inertia and rate-dependent material behaviour (Freund and Hutchinson, 1992).

The first analytical treatise of dynamic fracture was made by Mott, who amended the Griffith's energy balance for a central crack in an infinite plate with the kinetic energy of a fracture event. His modified expression for the strain energy release rate in an elastic continuum reads:

$$G(t) = \frac{dF}{da} - \frac{dU_E}{da} - \frac{dE_k}{da}, \tag{1}$$

where a is the crack length, F is the work done by external forces, U_E is the elastic strain energy given by:

$$U_E = U_{E_o} - \frac{\pi\sigma^2 a^2 B}{E}, \tag{2}$$

and E_K is the kinetic energy, given by:

$$E_k = \frac{1}{2}k^2\rho a^2\dot{a}^2\left(\frac{\sigma}{E}\right)^2. \tag{3}$$

In Eqs. (2) and (3), σ is the remotely applied stress normal to the crack, ρ and E are the density and Young's modulus of the material, $\dot{a}$ is the crack speed, and k is the wave constant. From Eqs. (1)–(3), Mott derived a time-dependent strain energy release rate (Mott, 1948):

$$G(t) = \frac{1}{2}\frac{d}{da}\left[\frac{\pi\ \sigma^2 a}{E} - \frac{k}{2}\rho a^2\dot{a}^2\left(\frac{\sigma}{E}\right)\right] = 2\Gamma \tag{4}$$

where Γ is a constant specific surface fracture energy. When compared to a material parameter, i.e. critical strain energy release rate G_c, Eq. (4) provides a criterion for crack stability: for $G(t) < G_c$ the crack will remain stationary, otherwise it will extend. This can be recast into a more familiar criterion based on comparison between

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(time-dependent) stress intensity factor, $K_I(t)$, and plane stain fracture toughness K_{Ic}, which is related to G_c via:

$$G_c = {K_{I_c}}^2\left(\frac{1-\nu^2}{E}\right). \tag{5}$$

Eq. (4) is derived with two limiting assumptions: the crack travels at a steady-state speed; and this speed is small compared to the shear wave speed within the material. However, due to the increased kinetic energy, dynamic fracture can occur below this critical limit for non-steady state speeds. Thus, K_I can be seen as a function of the crack velocity, which may not exceed a limiting value – the Rayleigh surface wave speed c_r. Kanninen and Popelar (1985) have shown that:

$$\dot{a} = c_r\left(1-\frac{K_{Ic}}{K_I}\right) \tag{6}$$

where $\dot{a}$ is the macro-crack speed. If K_{Ic} is assumed a material constant, i.e. fracture toughness is assumed independent of strain rate, then Eq. (6) will stand up to quantitative comparison with experimental data at low propagating speeds. Through extensive experiments on Homolite-100 by Ravi-Chandar and Knauss (1984), it was observed that a propagating crack does not exceed $\sim 0.7c_r$ due to multiple yet not fully explained dynamic fracture features which dissipate the fracture energy beyond this limit. They also observed the crack propagation process for a fast brittle crack contained a large diffuse zone of micro-cracks ahead of the tip. This process produced an oscillating macro-crack profile, slowing down the crack, leading in some cases to macro-crack branching (Ravi-Chandar and Knauss, 1984; Agwai et al., 2011).

The assumptions used by Mott for deriving Eq. (4) allow for two possible scenarios for modelling dynamic fracture:

- When a crack in a body subjected to a slowly varying load reaches a point of instability and propagates rapidly, leading to sudden unloading along a crack path. This is closer to a quasi-static situation, where the crack has a long time to dissipate energy relative to the fast propagation.
- When a body with a stationary crack is subjected to a rapidly varying load such as an impact, giving rise to high stress levels near the crack tip. This high stress level does not allow sufficient time for plastic deformations to develop before fracture, hence, energy must be dissipated by other mechanisms, e.g. micro-cracking. Therefore, energy is released within a short time frame leading to rapid crack propagation, possible sub-branching and or, macro-crack branching.

These two different scenarios have often been treated separately in fracture modelling due to the difficulty in integrating time-dependent and decaying discontinuities such as a crack in Fig. 1(a) into oscillating continuum systems under an external vibratory loading as in Fig. 1(b). This is because the strain waves produced by a propagating crack are often within the same order of magnitude as the global oscillating potential, making the resolution of a propagating crack within a model numerically stiff. While the separation of these scenarios is useful for analytical treatments, in reality they may be realised simultaneously and there is no reason to keep them separate when dynamic fracture is modelled numerically.

This paper offers a framework for numerical modelling of dynamic fracture where both scenarios are taken into account. As a first application of the framework, the dynamic crack propagation is followed up until the crack branching point, which is defined by the limit presented by Eq. (6). The post-branching behaviour is a subject of on-going work to be presented later. The developed modelling approach ensures energy conservation by allowing the energy released during crack propagation to be resolved by the global system through a quasi-explicit solver and a velocity dependent cohesive law implemented via the eXtended Finite Element Method. The strategy is tested on two well-established experiments and discussed in relation to other available approaches to modelling dynamic fracture. The first experimental comparison allows the crack to arrest before reflected strain waves interact. The second experimental comparison includes the interaction of reflected strain waves with the propagating crack, allowing for consideration of the effects of the interacting strain waves on a propagating crack.

2. Modelling

The proposed modelling approach has three components:

1. An implicitly treated velocity dependent ‘phenomenological’ cohesive law to represent the crack tip, implemented along the main crack path only.
2. A quasi-explicit solver to resolve the crack globally ensuring energy conservation.
3. A propagation algorithm using the eXtended Finite Element Method (XFEM) to represent the crack independent of a mesh.

The combination of these allows a propagating Fracture Process Zone (FPZ) to be integrated into a global continuum dynamic model and the energy from reflected waves to influence a propagating crack in an energetically conservative manner; effectively bypassing any numerical stiffness.

2.1. Phenomenological rate dependent cohesive zone

Cohesive zone models have been used in modelling dynamic fracture (Falk et al., 2001; Ferté, 2014; Camacho and Ortiz, 1996; Xu and Needleman, 1994) however, when used at every element boundary, they often lead to different results, particularly when the cohesive law contains elastic branch prior to damage initiation. This is because, the initial elastic traction-separation behaviour does not allow for resolving the cohesive zone without affecting the wave speed. To overcome this, Zhou et al. (2005) have suggested a more phenomenological crack-opening-rate-dependent cohesive law, which accounts for rate/velocity effects. This is introduced on the main crack path only, rather than for the micro-cracking process in the FPZ. The law has been derived from multiple experimental observations, summarised in Fig. 2. Specifically, irrespective of component geometry it has been observed that the dependence of Gc on the crack velocity, $\dot{a}_0$, is monotonically increasing and described with reasonable accuracy by a simple empirical expression (Areias and Belytschko, 2005):

$$G_c(\dot{a}_0) = G_0 \log\left(\frac{\dot{a}_L}{\dot{a}_L-\dot{a}_0}\right), \tag{7}$$

where $\dot{a}_L$ is the limiting crack velocity, and G_0 is the strain energy release rate at $\dot{a}_0 = 0$. The proposed equation is clearly an approximation to the real toughness–velocity relation at the two limits: Gc approached zero as crack velocity $\dot{a}_0$ approaches zero (i.e. the material is very brittle compared to fast fracture); Gc approached infinity as crack velocity approaches the limiting value. The rapid increase of Gc with $\dot{a}_0$ is explained with a velocity-toughening effect of the material (Tvergaard and Hutchinson, 1996).

To avoid the problem introduced by initial elastic traction-separation behaviour, we have considered a initially rigid-softening behaviour, schematically shown in Fig. 3, with toughness–velocity dependence based on Eq. (7). Since the critical stress is assumed to be independent of crack velocity, the illustrated behaviour is called opening-rate-dependent cohesive law. A fracture process zone with this law has been previously used to study crack branching (Crump et al., 2017).