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Dynamic stress intensity factor (Mode I) of a permeable penny-shaped crack in a fluid-saturated poroelastic solid

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ABSTRACT

A mathematical formulation is presented for the dynamic stress intensity factor (mode I) of a permeable penny-shaped crack subjected to a time-harmonic propagating longitudinal wave in an infinite poroelastic solid. In particular, the effect of the wave-induced fluid flow on the dynamic stress intensity factor is analyzed. The Hankel integral transform technique in conjunction with Helmholtz potential theory is used to formulate the mixed boundary-value problem as dual integral equations in the frequency domain. Using appropriate transforms, the dual integral equations can be reduced to a Fredholm integral equation of the second kind. The phenomenon of fluid flow along the crack surface has significant influences upon the frequency-dependent behavior of the dynamic stress intensity factor. The stress intensity factor monotonically decreases with increasing frequency, declining the fastest when the crack radius and the slow wave wavelength are of the same order. Such near-field information is of particular importance in predicting the crack strength subjected to oscillating loads. The characteristic frequency at which the stress intensity factor decays the fastest shifts to higher frequency values when the crack radius decreases.

1. Introduction

During recent decades, modeling mechanical properties of materials within the framework of poroelasticity has become increasingly important. This is mainly because many materials of practical interest are porous (Cheng, 2016) such as natural soils and rocks, and man-made concrete. Many studies have focused on the effects of wave-induced fluid flow (or fluid mass diffusion) due to the presence of mesoscopic cracks on elastic wave velocity dispersion (e.g., Hudson, 1981; Chapman, 2003; Brajanovski et al., 2005; Kong et al., 2013) and reflectivity (e.g., Barbosa and Rubino, 2016) in fluid-filled poroelastic media.

For the near-tip response of cracks in poroelastic solids subjected to dynamic loads previous investigations were restricted to impermeable cracks. Jin and Zhong (2002) determined the mode I dynamic stress intensity factor and obtained the solution for transient loads that were applied suddenly to the impermeable crack surface. Phurkhao (2013) investigated the mode I scattering problem but treated the case of an impermeable and traction-free crack. In his study, the transport of fluid across the crack surface is prohibited, and there are no stresses acting on the surface.

In contrast, we treat a permeable crack filled with an incompressible fluid. This paper analyses the dynamic stress intensity factor of a fluid-filled penny-shaped crack in an infinite porous medium when a plane, harmonic in time, longitudinal wave propagates perpendicularly to the surface of the crack. The analysis complements that of Galvin and Gurevich (2007) who investigated the effect of a permeable crack on the far-field scattering cross section at low frequencies. Because they were interested in the effective properties of fractured media in a seismology context they restricted their analysis to mesoscopic cracks, and low frequencies. They only focused on the far-field behavior of the scattering due to the use of Waterman-Truell multiple scattering theory (Waterman and Truell, 1961) to obtain effective medium properties. However, for a dynamic crack problem the far-field information is not useful in the sense that it offers no information for the development of the theories of crack propagation. In this study we use full-frequency Biot's theory of poroelastodynamics to model the mechanical behavior of the poroelastic material and investigate the near-tip field information.

The equations for poroelasticity as given by Biot (1956) can be written in different equivalent formulations (e.g., Biot, 1962; Bonnet, 1987). Bonnet (1987) has shown that the governing equa-

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tions of Biot's poroelastodynamics can be expressed in the $\mathbf{u}-p$, the displacement vector-pore fluid pressure, formulation without raising the order of the differential equations and that equations of poroelasicity and of thermoelasticity are mathematically equivalent in dynamic regime. In Biot (1962) the absolute fluid displacement vector $\mathbf{U}$ used in Biot (1956) is replaced by the relative fluid displacement vector $\mathbf{w}=\phi(\mathbf{U}-\mathbf{u})$, where ϕ is the porosity. Deresiewicz and Skalak (1963) have shown that the normal component of the displacement vector $\mathbf{w}$ is continuous on an open-pore interface between two dissimilar porous solids. This is due to the conservation of mass of the liquid at the interface. The continuity of the pore pressure is also required on the open-pore interface. This can be done with the aid of the constitutive equation (see Eq. (5) which relates the volumetric strain and increment of fluid content to the pore poressure).

It is worth mentioning again that the $\mathbf{u}-p$ and $\mathbf{u}-\mathbf{w}$ formulations are completely equivalent. To obtain the analytic solution of the wave scattering by a crack, we first apply the Helmholtz's potential theory to decompose the displacement vector $\mathbf{u}$ (Eq. (6)). By introducing amplitude ratios of relative fluid displacement and solid displacement in corresponding wave modes it is easy to express the relative fluid displacement vector $\mathbf{w}$ (see Eq. (7)). However, the $\mathbf{u}-p$ formulation drastically gives arise to the complexity of the mode decomposition making it be impractical to do analytically for the scattering problem. Therefore, we adopt Biot (1962)'s $\mathbf{u}-\mathbf{w}$ formulation which has been widely used to investigate the scattering by inclusions (Berryman, 1985; Gurevich et al., 1998; Liu et al., 2009) and wave propagation in layered structures (Gurevich and Lopatnikov, 1995).

The mathematical analysis of dynamic crack problems in a poroelastic solid usually follows the methods adopted for solving crack problems in the classical theory of elasticity. Sneddon and Lowengrub (1969) solved for the mode I stress distribution produced in an infinite elastic solid when a constant pressure or free surface boundary condition is applied over the surface of a penny-shaped crack. They formulated the elastostatic problem as a mixed boundary value problem in a half space and derived an expression for the stress intensity factor. Robertson (1967) investigated mode I scattering of a plane harmonic longitudinal wave by a penny-shaped crack in an infinite elastic solid. He used the Hankel integral transform approach to reduce the axisymmetric problem to dual integral equations and obtained an expression for the scattering cross-section. Mal (1968) solved the same wave problem to obtain an expression for the dynamic stress intensity factor. Moreover, many attempts have been made to solve the crack dynamic problems such as radial shear waves (e.g., Sih and Loeber, 1969) and torsional vibration (e.g., Sih and Loeber, 1968; Mal, 1970), scattering associated with anisotropy (e.g. Kundu, 1990; Kundu and Boström, 1992), obliquely incident waves (e.g., Piau, 1979; Angel and Achenbach, 1985), and numerical approaches (e.g., Zhang and Gross, 1992).

In this paper, we analytically study the elastic wave scattering by a fluid-filled crack in a poroelastic medium. In particular, the steady-state dynamic stress intensity factor is derived. The background porous medium is assumed to be governed by Biot's equations of dynamic poroelasticity, while the mesoscopic crack is assumed to be much larger than the size of micropores. We assume that the thickness of the crack is much smaller than the wavelength of the incident longitudinal wave. To obtain the stress and pore pressure fields near the tip of a crack, the poroelastodynamics problem is solved by using the Hankel integral transform. Thereafter, the mixed-boundary value problem is reduced to a Fredholm integral equation of the second kind. As is shown in this paper the wave-induced fluid flow has a significant effect on the frequency-dependent behavior of the dynamic stress intensity factor.

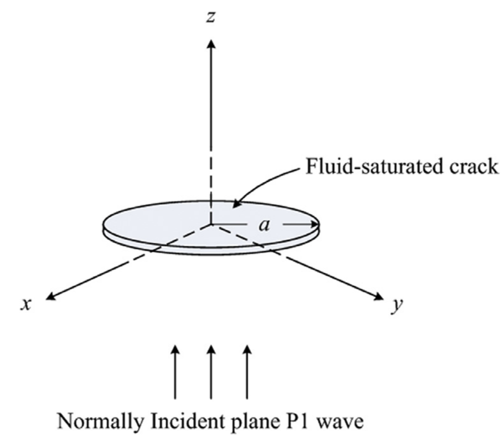

Fig. 1. Fluid-saturated penny-shaped crack in a poroelastic solid: Mode I problem.

2. Problem formulation

2.1. Equations of Biot's porodynamics

Consider an infinite isotropic, fluid-saturated poroelastic solid that is homogeneous except for a flat penny-shaped crack of radius a. A cylindrical coordinate system (r, θ, z) with the origin at the center of the crack is introduced. The internal crack which is saturated by a fluid occupies the region $z=0$, $0 \le r \le a$. The problem is to determine the scattering of an incident plane longitudinal wave, harmonic in time, propagating in the positive direction of the z-axis of a cylindrical coordinate system. The incident P-mode wave is a longitudinal wave of the first kind (also referred to as P1-wave) which is described by Biot's theory of poroelastodynamics (Biot, 1956). Fig. 1 illustrates a schematic diagram of the problem considered in this article.

The time-harmonic factor $e^{-\mathrm{i}\omega t}$, where ω is the circular frequency, is suppressed throughout. The equations of motion expressed in the frequency domain are (Biot, 1962)

$$\nabla \cdot \boldsymbol{\sigma} = -\omega^2\left(\rho\mathbf{u} + \rho_f\mathbf{w}\right), \tag{1}$$

$$\nabla p = \omega^2\left(\rho_f\mathbf{u} + \tilde{\rho}\mathbf{w}\right), \tag{2}$$

where $\boldsymbol{\sigma}$ and p are the total stress tensor and pore fluid pressure, the vector $\mathbf{u}$ is the solid displacement, the vector $\mathbf{w}$ represents the displacement of the fluid flowing relative to the solid but measured in terms of volume per unit area of the bulk medium, ρ_f is the pore fluid density. $\rho=(1-\phi)\rho_s+\phi\rho_f$ is the density of the overall medium, where ϕ is the porosity, ρ_s is the solid density. $\tilde{\rho} = \frac{\mathrm{i}\eta}{\omega\kappa(\omega)}$ is the effective filtration density, where $\mathrm{i}=\sqrt{-1}$, η is the dynamic viscosity of pore fluid, $\kappa(\omega)$ is the so-called dynamic permeability. According to Johnson et al., (1987), the dynamic permeability can be taken as static permeability κ_0 multiplied by its frequency correction factor

$$\kappa(\omega) = \kappa_0\left[\sqrt{1-\mathrm{i}\frac{\omega}{2\omega_{\mathrm{B}}}} - \mathrm{i}\frac{\omega}{\omega_{\mathrm{B}}}\right]^{-1}, \tag{3}$$

where the frequency $\omega_{\mathrm{B}} = \frac{\phi\eta}{\kappa_0\alpha_\infty\rho_f}$ is the so-called Biot critical frequency (Biot, 1956) which separates the viscous-force dominated flow from the inertial-force-dominated flow, α_∞ is the tortuosity. The constitutive relationships between the displacements and the stresses are (Biot and Willis, 1957)

$$\boldsymbol{\sigma} = [(H-2\mu)\nabla\cdot\mathbf{u} + C\nabla\cdot\mathbf{w}]\mathbf{I} + \mu\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right], \tag{4}$$

$$-p = C\nabla\cdot\mathbf{u} + M\nabla\cdot\mathbf{w}, \tag{5}$$