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Discrete equivalent wing crack based damage model for brittle solids

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ABSTRACT

The Discrete Equivalent Wing Crack Damage (DEWCD) model formulated in this paper couples micromechanics and Continuum Damage Mechanics (CDM) principles. At the scale of the Representative Elementary Volume (REV), damage is obtained by integrating crack densities over the unit sphere, which represents all possible crack plane orientations. The unit sphere is discretized into 42 integration points. The damage yield criterion is expressed at the microscopic scale: if a crack is in tension, crack growth is controlled by a mode I fracture mechanics criterion; if a crack is in compression, the shear stress that applies at its faces is projected on the directions considered in the numerical integration scheme, and cracks perpendicular to these projected force components grow according to a mode I fracture mechanics criterion. The projection of shear stresses into a set of tensile forces allows predicting the occurrence of wing cracks at the tips of pre-existing defects. We assume that all of the resulting mode I cracks do not interact, and we adopt a dilute homogenization scheme. A hardening law is introduced to account for subcritical crack propagation, and non-associated flow rules are adopted for damage and irreversible strains induced by residual crack displacements after unloading. The DEWCD model depends on only 6 constitutive parameters which all have a sound physical meaning and can be determined by direct measurements in the laboratory. The DEWCD model is calibrated and validated against triaxial compression tests performed on Bakken Shale. In order to highlight the advantages of the DEWCD model over previous anisotropic damage models proposed for rocks, we simulated: (a) A uniaxial tension followed by unloading and reloading in compression; and (b) Uniaxial compression loading cycles of increasing amplitude. We compared the results obtained with the DEWCD model with those obtained with a micro-mechanical model and with a CDM model, both calibrated against the same experimental dataset as the DEWCD model. The three models predict a non linear-stress/strain relationship and damage-induced anisotropy. The micro-mechanical model can capture unilateral effects. The CDM model can capture the occurrence of irreversible strains. The DEWCD model can capture both unilateral effects and irreversible strains. In addition, the DEWCD model can predict the apparent increase of strength and ductility in compression when the confinement increases and the increasing hysteresis on unloading-reloading paths as damage increases. The DEWCD model is the only of the three models tested that provides realistic values of yield stress and strength in tension and compression. This is a significant advancement in the theoretical modeling of brittle solids. Future work will be devoted to the prediction of crack coalescence and to the modeling of the material response with interacting micro-cracks.

1. Introduction

In most brittle materials such as rocks, concrete and ceramic composites, mechanical failure is the result of a sequence of coupled micro-processes. In Continuum Damage Mechanics (CDM), anisotropic damage is usually represented by second-order tensors (Murakami, 1988; Halm and Dragon, 1996) or fourth-order tensors (Ju, 1989) that depend on the density and orientation of families of micro-cracks. The expression of the damaged stiffness tensor is based on the principle of strain or energy equivalence (Murakami, 2012), and stress/strain relationships are deduced from the thermodynamic relationships that are derived from the energy potentials. The damage flow rule, combined with the consistency condition, allows determining the evolution of the magnitude and direction of micro-cracks (Simo and Ju, 1987; Chaboche, 1993; Hayakawa and Murakami, 1997). CDM models were implemented in Finite Element Methods (FEM) for practical engineer-

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ing purposes (e.g., Jin et al. (2015); Xu and Arson (2015); Jin et al. (2016)) and were successfully used to predict damage-induced anisotropy and confinement-induced strengthening in rock subject to compression (e.g., Shao and Rudnicki (2000); Shao et al. (2005); 2006)), as well as unilateral effects (e.g., Chaboche (1993); Dragon et al. (2000)). However, multiple non-linear damage phenomena require more constitutive parameters that are often not related to any microstructure or mechanical property, which raises calibration challenges (Halm and Dragon, 1996; 1998; Arson, 2014). Moreover, difficulties arise when distinguishing tension and compression: either the stress or the strain tensor has to be split into positive and negative components. Damage evolution depends on distinct yield criteria and damage potentials (Lubarda et al., 1994; Frémond and Nedjar, 1996; Comi and Perego, 2001; Zhu and Arson, 2013). These so called bi-dissipative models are based on complex mathematical formulations (challenging to implement in FEM) and depend on a large number of parameters (challenging to calibrate). In micromechanical models, a direct relationship is established between the macroscopic mechanical behavior and micro-crack initiation, propagation, opening, closure and frictional sliding. In the dilute crack scheme, the calculation of the displacement jump across crack faces (Budiansky and O'connell, 1976) is used as a basis to upscale the effective properties of the damaged REV (Kachanov, 1992; 1993) and to express the corresponding energy potentials (Kachanov, 1982a; 1982b; Pensée et al., 2002; Pensee and Kondo, 2003). The evolution law is based on fracture mechanics and can represent Mode I splitting (Krajcinovic et al., 1991; Gambarotta and Lagomarsino, 1993), Mode II friction sliding (Gambarotta and Lagomarsino, 1993) or mixed Mode wing crack development (Kachanov, 1982b; Nemat-Nasser and Obata, 1988). In order to account for crack interactions, one can explicitly express the stress field that results from external loading and crack interaction (Paliwal and Ramesh, 2008). Other upscaling techniques (e.g., Zhu et al. (2008); 2009); Zhu and Shao (2015); Qi et al. (2016a); 2016b)) resort to Eshelby homogenization procedure (Eshelby, 1957), in which the cracked solid is viewed as a matrix-inclusion system (Dormieux et al., 2006). Micromechanical formulations automatically predict unilateral effects but usually cannot capture the inelastic deformation together with the softening that characterize the REV behavior after the peak of stress, and they require computationally intensive resolution algorithms. In this paper, we formulate an anisotropic damage model that couples micro-mechanical crack propagation criteria and CDM energy principles with a minimum number of constitutive parameters. In Section 2, we present the theoretical formulation of our model, called the Discrete Equivalent Wing Crack based Damage model (DEWCD). A finite number of orientations is used to project the normal and tangential crack displacement vectors. The damage variable is a second-order crack density tensor, and the irreversible deformation is the crack opening vector averaged over all possible crack orientations. In tension, cracks propagate in mode I in the direction normal to the tensile stress. In compression, wing cracks propagate in mode I in the direction of the minimum deviatoric stress. We calibrate and validate the DEWCD model against triaxial compression data obtained on Middle Bakken shale. In Section 3, we use the same experimental dataset to calibrate a phenomenological damage model, the Differential-Stress Induced Damage (DSID) model (Xu and Arson, 2014; 2015) and a micromechanical damage model (Pensée et al., 2002; Pensee and Kondo, 2003). We simulate: (1) A uniaxial tension followed by unloading and uniaxial compression; and (2) Two loading-unloading cycles of uniaxial compression of increasing amplitude. We compare the performance of the three models for capturing damage-induced anisotropy of stiffness, unilateral effects in compression, damage hysteresis during unloading-reloading cycles, damage-induced irreversible strains, confinement-dependent strength, and differences of behavior in tension and compression.

2. Theoretical formulation of the discrete equivalent wing crack damage (DEWCD) model

2.1. Micromechanics-based free enthalpy

We formulate a CDM model in which the expression of the free enthalpy is obtained from micromechanics principles. In the following, we consider a REV of volume Ω_r and external boundary $\partial\Omega_r$, in which a large number of penny shaped microscopic cracks of various orientations are embedded in an isotropic linear elastic matrix of compliance tensor $\mathbb{S}_0$. Each microscopic crack is characterized by its normal direction $\vec{n}$ and its radius a, which is at least 100 times smaller than the REV size. Opposite crack faces are noted ω^+ and ω^-, with normal vectors $\vec{n}^+$ and $\vec{n}^-$. The displacement jump is noted:

$$[\vec{u}\,] = \vec{u}^{\,+} - \vec{u}^{\,-} \tag{1}$$

Where $\vec{u}^{\,+}$ (respectively $\vec{u}^{\,-}$) denotes the displacement vector at face ω^+ (respectively ω^-). We consider a uniform stress field $\boldsymbol{\sigma}$ applied at the boundary $\partial\Omega_r$. The displacement field at the REV scale is calculated by superposition, by adding up the displacement field in the elastic matrix in the absence of cracks and the displacement field induced by the opening and sliding of micro-crack faces.

We assume that the mechanical interaction between cracks is negligible and we use a dilute homogenization scheme to calculate the crack displacement jumps. As a result, the average micro stress is equal to the stress field applied to the REV, so that we have:

$$\boldsymbol{\sigma} = \frac{1}{|\Omega_r|}\int_{\Omega_r}[\boldsymbol{\sigma}^m(\boldsymbol{x}) + \boldsymbol{\sigma}^c(\boldsymbol{x})]\mathrm{d}\boldsymbol{x} \tag{2}$$

In which $\boldsymbol{\sigma}^c$ is the stress field that is applied at micro-crack faces and $\boldsymbol{\sigma}^m$ is the stress field in the linear elastic matrix. Moreover, the local stress at crack faces is self-equilibrating, so that:

$$\frac{1}{|\Omega_r|}\int_{\Omega_r}\boldsymbol{\sigma}^c(\boldsymbol{x})\mathrm{d}\boldsymbol{x} = 0 \tag{3}$$

And therefore:

$$\boldsymbol{\sigma} = \langle\boldsymbol{\sigma}^m\rangle \tag{4}$$

The strain tensor in the matrix is obtained as follows:

$$\boldsymbol{\epsilon}^m = \mathbb{S}_0 : \boldsymbol{\sigma}. \tag{5}$$

Each micro-crack can be considered as a single crack embedded in an infinite elastic homogeneous matrix, which allows calculating the displacement jumps from fracture mechanics principles (Horii and Nemat-Nasser, 1983; Kachanov et al., 2013). Considering a penny shaped crack of radius a subjected to a uniformly distributed normal stress p at its faces and embedded in an infinite elastic medium with Young's modulus E_0 and Poisson's ratio ν_0, the normal displacement jump is:

$$[u_n] = 8\frac{1-\nu_0^2}{\pi E_0}p\sqrt{a^2 - r^2} \tag{6}$$

The corresponding average Crack Opening Displacement (COD) is therefore:

$$\Big\langle [u_n] \Big\rangle = \frac{16}{3}\frac{1-\nu_0^2}{\pi E_0}pa \tag{7}$$

Similarly, considering a penny shaped crack of radius a subjected to a uniformly distributed shear stress $\vec{\tau}$ at its faces and embedded in an infinite elastic medium with Young's modulus E_0 and Poisson's ratio ν_0, the shear displacement jump is expressed as (Kachanov et al., 2013):

$$\Big\langle [\vec{u_t}\,] \Big\rangle = \frac{32}{3}\frac{1-\nu_0^2}{(2-\nu_0)\pi E_0}\vec{\tau}\,a \tag{8}$$