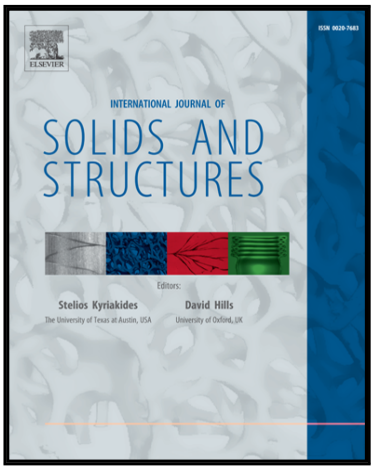

Gradient-elastic stress analysis near cylindrical holes in a plane under bi-axial tension fields

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Gradient-elastic stress analysis near cylindrical holes in a plane under bi-axial tension fields

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Abstract

This article is devoted to a gradient-elastic stress analysis of an infinite plate weakened by a cylindrical hole and subjected to two perpendicular and independent uni-axial tensions at infinity. The problem setting can be considered as an extension and generalization of the well-known Kirsch problem of the classical elasticity theory which is here extended with respect to the external loadings and generalized with respect to the continuum framework. A closed-form solution in terms of displacements is derived for the problem within the strain gradient elasticity theory on plane stress/strain assumptions. The main characters of the total and Cauchy stress fields are analyzed near the circumference of the hole for different combinations of bi-axial tensions and for different parameter values. For the original Kirsch problem concerning a uni-axially stretched plate, the analytical solution fields for stresses and strains are compared to numerical results. These results are shown to be in a full agreement with each other and, in particular, they reveal a set of new qualitative findings about the scale-dependence of the stresses and strains provided by the gradient theory, not common to the classical theory. Based on these findings, we finally consider the physicalness of the concepts total and Cauchy stress appearing in the strain gradient model.

Strain gradient elasticity, Kirsch problem, Cauchy stress, total stress, plane stress/strain problem

1 Introduction

The fundamental knowledge of the mechanical behavior of solids has been successfully accumulated by studying certain applied model problems of the classical theory of elasticity, e.g., the Lamé problem, the Kirsch problem and the Eshelby's inclusion problem. In particular, analytically derived stress and strain fields allow one to investigate both the qualitative and the quantitative material behavior of solids such as predicting locations of stress concentrations and further failure for brittle materials, or plasticity initiation for ductile materials.

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