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Collapse of liquid-overfilled strain-isolation substrates in wearable electronics

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ABSTRACT

Liquid that resides in a soft elastomer embedded between wearable electronics and biological tissue provides a strain-isolation effect, which enhances the wearability of the electronics. One potential drawback of this design is vulnerability to structural instability, e.g., roof collapse may lead to partial closure of the liquid-filled cavities. This issue is addressed here by overfilling liquid in the cavities to prevent roof collapse. Axisymmetric models of the roof collapse are developed to establish the scaling laws for liquid-overfilled cavities, as well as for air- and liquid-filled ones. It is established that the liquid-overfilled cavities are most effective to prevent roof collapse as compared to air- and liquid-filled ones.

1. Introduction

Soft (Cheng et al., 2011; Kim et al., 2009; Wu et al., 2010) or stiff substrates (Robinson et al., 2014; Romeo et al., 2013), embedded between wearable electronics (Chen et al., 2016; Chortos et al., 2016; Webb et al., 2015; Xu and Zhu, 2012) and biological tissues, can shield the electronics from strains induced by the biological tissue, i.e., they can provide “strain isolation”. The soft substrates also shield the biological tissues from sensing the existence of the wearable electronics (Koh et al., 2016; Lee et al., 2015). A new design (Ma et al., 2017) introduces cavities in the substrate in order to enhance the strain-isolation effect. One potential drawback of this design is that the cavities may close due to adhesion (the so called “roof collapse” (De Boer and Michalske, 1999; Huang et al., 2005; Mastrangelo and Hsu, 1993)), which eliminates the desired strain-isolation effects. Ma et al. (2017) proposed injecting liquid into the cavities to prevent roof collapse, and they also developed two-dimensional models for air-filled and liquid-filled cavities; however, a 2D model may significantly overestimate roof collapse (Xue et al., 2017).

A more effective method to overcome roof collapse is proposed in this paper by overfilling liquid in the cavities. For circular cavities implemented in the experiments, axisymmetric models are more accurate than 2D ones, and are thus developed for liquid-overfilled cavities, as well as air- and liquid-filled ones. A scaling law is developed to give the critical work of adhesion, below which roof collapse will not occur. This critical work of adhesion depends strongly on the amount of liquid overfill.

2. Analytic model of collapse

Fig. 1 shows the schematic illustrations of roof collapse of circular air-filled (Fig. 1a and b), liquid-filled (Fig. 1c and d) and liquid-overfilled (Fig. 1e and f) cavities (radius R and height h) in the substrate, with the collapse radius αR to be determined. The layers above and below the cavity are called the top layer and bottom layer of the substrate, which is sandwiched between the circular electronic device (radius κR, thickness t_1, modulus E_1, and Poisson's ratio ν_1) and the biological tissue. Roof collapse of the cavity occurs mainly due to the deformation of the top layer (thickness t_2, modulus E_2, and Poisson's ratio $\nu_2 = 0.5$ for most elastomers)

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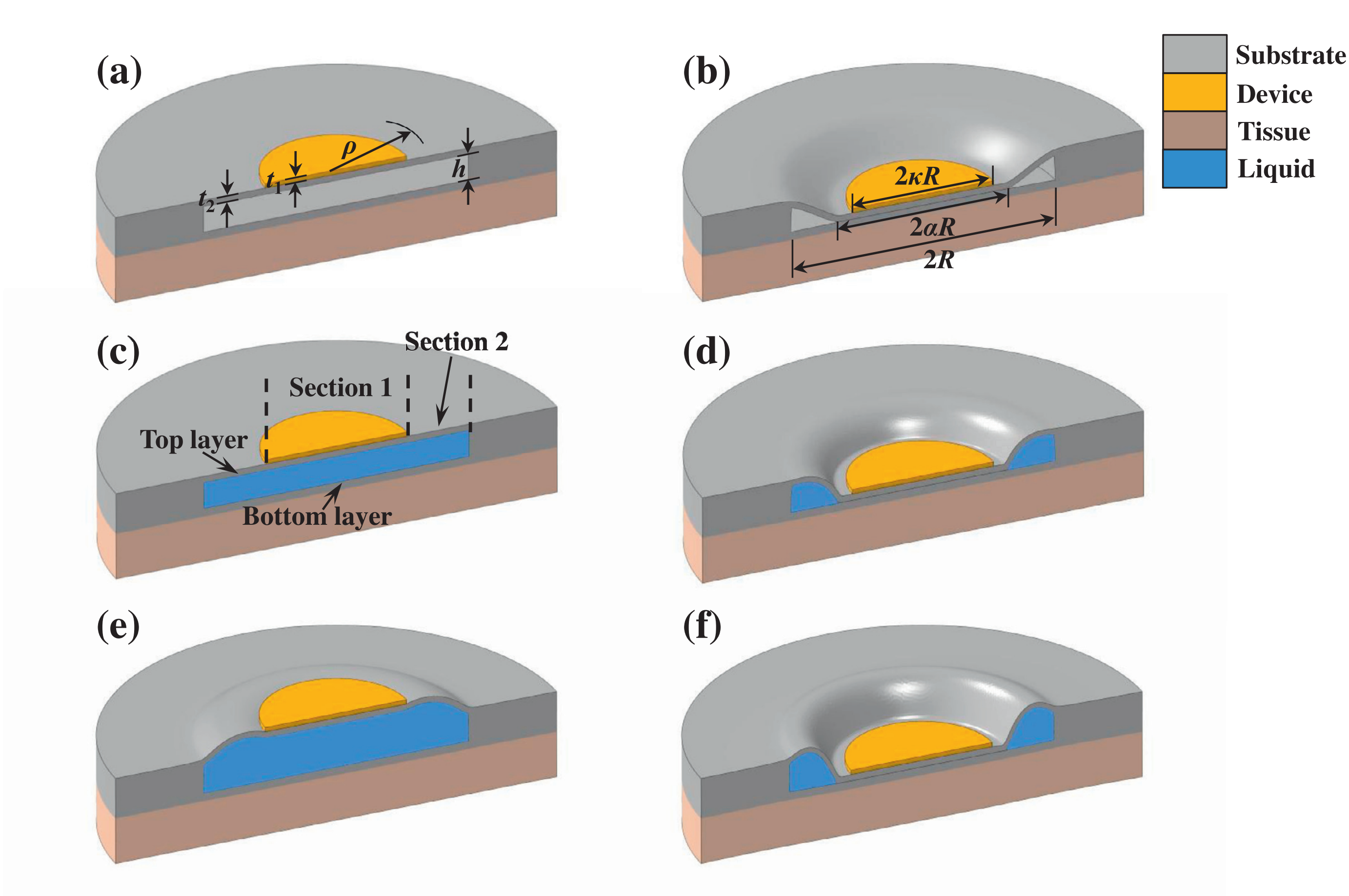

Fig. 1. Cross-sectional schematic illustrations of **(a, c, e)** the circular cavities and **(b, d, f)** their collapsed states **(a, b)** filled with air, **(c, d)** filled with liquid, and **(e, f)** overfilled by liquid.

(Ma et al., 2017). With the stress-free state (Fig. 1a and c) defined as the ground state (i.e., zero energy), the total potential energy due to roof collapse is (Huang et al., 2005)

$$U_{total} = U_{deformation} - \pi\alpha^2 R^2 \gamma, \tag{1}$$

where $U_{deformation}$ is the deformation energy of the top layer and the device (and the deformation energy of the bottom layer is negligible (Ma et al., 2017)), and γ is the work of adhesion between the top and bottom surfaces of the cavity. The device and top layer are modeled as plates because their thicknesses are much less than their radius, $t_1 < <R$ and $t_2 < <R$. The normalized total potential energy in Eq. (1) then takes the form

$$\frac{R^2 U_{total}}{Dh^2} = \frac{R^2 U_{deformation}}{Dh^2} - \pi\gamma'\alpha^2, \tag{2}$$

where $\gamma' = \gamma R^4/Dh^2$ is the normalized work of adhesion.

2.1. Air-filled cavity

The deflection w is -h in the collapsed region ($0 \le \rho \le \alpha R$, where ρ is the polar coordinate, Fig. 1a). For an air-filled cavity, the deflection w in the uncollapsed region ($\alpha R < \rho \le R$) satisfies the equilibrium equation in polar coordinates (Blaauwendraad, 2010),

$$\left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho}\right)\left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho}\right)w = 0. \tag{3}$$

The boundary conditions are

$$w\big|_{\rho=\alpha R} = -h, \tag{4a}$$

$$\frac{dw}{d\rho}\Big|_{\rho=\alpha R} = 0, \tag{4b}$$

$$w\big|_{\rho=R} = 0, \tag{4c}$$

$$\frac{dw}{d\rho}\Big|_{\rho=R} = 0. \tag{4d}$$

The deformation energy in the collapsed region ($0 \le \rho \le \alpha R$) is zero because its deflection keeps a constant (w ==-h). Therefore, the deformation energy of the whole region ($0 \le \rho \le R$) is obtained from w by the plate theory as (Timoshenko and Woinowsky-Krieger, 1959)

$$U_{deformation} = \int_{\alpha R}^{R} \left\{ D'_{11}\left[\left(\frac{\partial^2 w}{\partial\rho^2}\right)^2 + \left(\frac{1}{\rho}\frac{\partial w}{\partial\rho}\right)^2\right] + 2D'_{12}\frac{\partial^2 w}{\partial\rho^2}\left(\frac{1}{\rho}\frac{\partial w}{\partial\rho}\right)\right\}\rho\pi\,\mathrm{d}\rho, \tag{5}$$

where D'_{11} and D'_{12} are the bending stiffness; $D'_{11} = D = E_2 t_2^3/[12(1-\nu_2^2)]$ and $D'_{12} = \nu_2 D'_{11}$ ($\nu_2 = 0.5$) for Section 2 ($\kappa R < \rho \le R$) in Fig. 1c that does not have the device (i.e., top layer only), and $D'_{11} = mD$ and $D'_{12} = \eta mD$ for Section 1 ($0 \le \rho \le \kappa R$) in Fig. 1c consisting of both the top layer and the device, with m and η given in terms of the plane-strain moduli $\bar{E}_1 = E_1/(1-\nu_1^2)$ and $\bar{E}_2 = E_2/(1-\nu_2^2)$ as (Daniel et al., 1994) (See Appendix A