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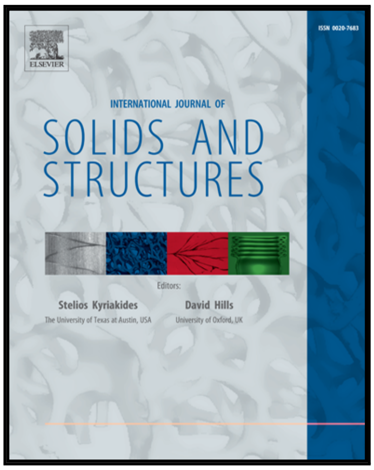

Second-order linear plate theories: Partial differential equations, stress resultants and displacements

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Second-order linear plate theories:
Partial differential equations, stress resultants and displacements

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Keywords uniform-approximation technique, pseudo-reduction approach, consistent plate theory, second-order theory, stress resultants, displacements

Abstract
By combining the uniform-approximation technique with the pseudo-reduction approach, a consistent second-order plate theory is developed with recourse to neither kinematical assumptions nor to shear-correction factors by truncation of the elastic energy. The governing partial differential equations and the expressions for the stress resultants are compared with those of other authors. Free coefficients of the resulting displacement “ansatz” are determined a posteriori, in order to satisfy three-dimensional boundary conditions and local equilibrium equations.

1 Introduction
Plates are thin plane structures loaded transversally to their midplanes. The characteristic thickness dimension h is much smaller than the characteristic in-plane dimension a, $h \ll a$. Plate theories are inherently approximative in that they attempt to describe the actual three-dimensional plate continuum by quantities that are defined on a surface. Generally, three different categories of derivation techniques for plate theories may be distinguished. The *classical* or *engineering approach* starts with a set of kinematical a priori assumptions for the stress and displacement distributions in thickness direction. Either transvers shear strains are neglected, or their influence is considered by the introduction of shear-correction factors. A historical survey on classical plate theories may be found, e.g., in [1].

Following the *direct approach*, all quantities “live” on a Cosserat-type surface endowed with a set of deformable directors attached at each point of the plane. Despite of the mathematical elegance, the main drawback of the method lies in the problem of establishing constitutive relations. Material parameters are identified by comparisons with a set of solutions of known test problems. The choice of test problems has a crucial influence on the resulting theory. An excellent overview over the theories relying on the direct approach is given in [2] with an extended bibliography.

Within the present paper, we will follow the *consistent approach* introduced in the pioneering treatise [3] by Naghdi. Starting from the three-dimensional theory of elasticity, the approach uses abstract Fourier-series expansions in thickness direction with respect to a suitable basis to achieve a dimension reduction. The basis might be monomials, scaled Legendre polynomials or trigonometric functions. After introducing proper non-dimensional quantities and integrating over the plate thickness, the dimensionless plate parameter $c^2 = h^2/(12a^2)$ evolves quite naturally, which is a small quantity in plate theories. The elastic potential, i.e., the sum of the strain-energy and the potential of external forces, both per unit of plate area, appear as power series in this plate parameter. This infinite series can be truncated at different