International Journal of Solids and Structures 115–116 (2017) 27–42

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A complete three-dimensional continuum model of wing-crack growth in granular brittle solids

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ARTICLE INFO

Article history:
Received 17 November 2015
Revised 3 February 2017
Available online 17 February 2017

Keywords:
Brittle failure
Sliding crack
Wing crack
Stick slip
Anisotropic damage
Ice failure

ABSTRACT

Failure of brittle materials containing embedded three dimensional pre-cracks and subjected to uniaxial compressive and tensile loading is considered here. The sliding crack (wing-crack) model of Ashby and Sammis (1990) is extended and further developed to formulate a 3D anisotropic continuum damage model.

First, a frictional sliding condition of pre-cracks is formulated in three dimensions and a crack interaction function is proposed. To introduce inelastic strains due to cracking, crack opening displacements are derived from Castigliano's second theorem. Finally, a strain–stress relation is obtained from the Gibbs energy density equation.

The model was implemented in Abaqus/Explicit finite element software. Material inhomogeneity was considered assuming that the pre-cracks are lognormally distributed between integration points.

While testing the proposed model against experimental results of granular ice, the numerical simulations were in good agreement both under uniaxial compression and tension as a function of grain size and temperature-dependent kinetic friction. The model was able to predict qualitatively and quantitatively the brittle failure modes and strength both under compression and under tension. Due to the modelled inhomogeneity, the scatter in simulated strengths corresponded to that of the test results. Besides non-simultaneous and non-uniform damaging, the model revealed important phenomena observed during the experiments; e.g. under compression the sliding of the pre-cracks resembled "stick-slip" motion, and secondary cracks were observed to grow in a jerky manner. The effect of specimen end conditions on both the failure stress and failure mode was addressed in the simulations.

1. Introduction

One of the greatest challenges of material failure analysis is the modelling of brittle failure. Materials like natural ice and rock are heterogeneous and crystalline, containing pores and flaws and other weaknesses. These materials exhibit ductile behaviour under high confinement, and brittle behaviour under low confinement (Nemat-Nasser and Horii, 1984; Renshaw and Schulson, 2001; Wachter et al., 2009; Wong and Baud, 2012). When these materials are subjected to uniaxial compressive loading in the brittle regime, they are known to fail by axial splitting along the loading direction (Horii and Nemat-Nesser, 1985; Renshaw and Schulson, 2001). Formation, growth and interaction of (micro)cracks due to material inhomogeneities and external force are considered to be the mechanism of brittle failure in uniaxial compression (Brace and Bombolakis, 1963; Fairhurst and Cook, 1966; Horii and Nemat-Nasser, 1986; Nemat-Nasser and Horii, 1984; Renshaw and Schulson, 2001).

1.1. Two-dimensional sliding crack model

One of the mechanisms proposed for modelling of axial splitting is the sliding crack or 'wing crack' model (Ashby and Hallam, 1986; Brace and Bombolakis, 1963; Fairhurst and Cook, 1966; Horii and Nemat-Nasser, 1986; Renshaw and Schulson, 2001). The model was originally proposed for a 2D through crack in a plate. As illustrated in Fig. 1a, the splitting failure begins when a primary crack (pre-crack) undergoes sliding, creating secondary cracks (wings) at the tips of the primary crack. The macroscopic failure occur when a series of cracks extend and finally link together and split the material. The wedged crack can be modelled as a straight representative crack, where the wedging forces on the pre-crack area are opening the crack. This is often called the Fairhurst-Cook model

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http://dx.doi.org/10.1016/j.ijsolstr.2017.02.012

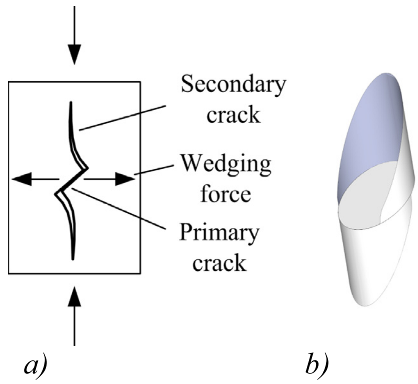

Fig. 1. Sliding cracks. (a) 2D crack; (b) wrapping of single 3D crack under uniaxial compression.

(1966). The model has been verified by several series of successful tests (e.g. Brace and Bombolakis, 1963; Iliescu and Schulson, 2004; Nemat-Nasser and Horii, 1982). A review of the most common 2D wing crack models is presented in the paper by Lehner and Kachanov (1996).

When a specimen is subjected to confining compression, the failure mode changes gradually from brittle to ductile under increasing confinement. Under uniaxial loading the axial splits are nearly vertical, but under confined compression the wings tend to form an inclined band and the mode of macroscopic failure is shear-like. In an analysis of uniaxial compressive failure in 2D, Horii and Nemat-Nasser (1986) found that depending on the ratio of confining pressure to most compressive stress, the macroscopic failure mode is either shear faulting or ductile flow. It appears that the lengths of interacting wing cracks under confined compression are shorter than those of wing cracks under uniaxial compression (Horii and Nemat-Nasser, 1986). In addition to the linkage of cracks formed prior to faulting, propagation of shear faulting can be accompanied by initiation of new cracks (Wachter et al., 2009).

1.2. Three-dimensional sliding crack

If disk-shaped primary cracks are embedded into a 3D medium, the continuum response and behaviour depends on the stress state, number, spacing and orientation of primary cracks as described here.

If a single disc shaped 3D primary crack is under uniaxial loading, the secondary crack is known to wrap (curl) around the primary crack as illustrated in Fig. 1b (Adams and Sines, 1978; Cannon et al., 1990; Dyskin et al., 1994b). Dyskin et al. (1994b) observed that the growth of secondary cracks was stable until the length of the wrapped wings was 1.0–1.5 times the diameter of the primary crack. Under further loading the wings did not grow, and there followed burst-like failure of the sample. Dyskin et al. (1994b) concluded that a single embedded crack cannot extensively grow under uniaxial compression.

Sahouryeh et al. (1998, 2002) studied experimentally a single 3D primary crack under biaxial compression using specimens fabricated from resin, sandstone and concrete. Due to lateral confinement, wrapping (curling) of the secondary crack was prevented and the resulting failure mode was splitting along the loading directions. Growth of the secondary crack formed a disk-like crack as illustrated in Fig. 3a.

When several cracks are embedded into a 3D medium, the behaviour under uniaxial compression changes due to interaction between the cracks. Dyskin et al. (2003) studied experimentally the effect of shape, spacing, orientation and location of primary cracks under uniaxial compression of a polyester resin. In all cases the samples "almost split into columns." In samples with multiple primary cracks, the details of crack growth could not be traced visually because of the large concentration of cracks obscuring visual observation. They concluded that the presence of a multitude of cracks provides a self-sufficient mechanism of failure. The interactions of cracks produce new cracks, which can grow substantially and split the specimen along vertical planes (Dyskin et al., 2003).

Although sliding crack models have been applied in two dimensions, consistent 3D continuum models have not been proposed and implemented in numerical software such that both the crack growth mechanism and the anisotropic nature of the micro mechanism were considered in the models, although 3D models motivated by the sliding crack mechanism have been recently developed and proposed (Bhat et al., 2011; Kolari, 2013; Pensee et al., 2002; Shao et al., 2006; Swoboda and Yang, 1999; Yu and Feng, 1995). Dyskin et al. (1999) proposed a 3D model where the crack is modelled as a disk-like crack growing under compression driven by wedging forces in the middle of the disk. They consider both the initial stage of growth of secondary cracks and the latest stages of loading. The interaction of cracks as a function of location and spacing was studied also. They conclude that two horizontally aligned inclined cracks (coplanar or perpendicular) can interact and lead to the appearance of a large splitting crack instead of a wrapping crack. When the distance between crack centres was less than four times the crack radius, the stress concentration was found to be quite pronounced. Sahouryeh et al. (2002) concluded that an inclined crack under biaxial compression can be modelled as a disk-like crack loaded by opposite parallel forces. They proposed a model that takes into account the distance of a disk-like crack from the boundary of a specimen.

2. Description of the problem

Continuum modelling of brittle failure of a material containing embedded pre-cracks is considered in this paper. The uncracked material is assumed to be homogeneous and isotropic. The fundamental objective is to formulate the effect of microscopic frictional sliding and crack growth on macroscopic inelastic deformations in three dimensions.

2.1. Objectives and scope

The primary objective is to apply and extend current knowledge of the wing crack mechanism by introducing a consistent continuum mechanics formulation in three dimensions. The objectives are to capture the following features:

1. Crack growth based on the sliding crack mechanism.
2. Axial splitting failure mode under uniaxial compression.
3. Transverse cracking failure mode under tension.
4. Cracking (damaging) induced anisotropy.
5. The unilateral condition (crack closure).
6. Interaction of cracks.

The model is implemented with FE software; therefore the derivation and formulation should be suitable for FE implementation. The objective is also to introduce the material inhomogeneity into the FE implementation.

The model is based on the 3D approach proposed by Ashby and Sammis (1990). Compared to conventional wing crack models, the major improvements to the model proposed here are: (a) cracks are not assumed to be parallel, and size and orientation are based solely on the state of stress; (b) damage-induced anisotropy is introduced; (c) the tensile failure mode is included in the formulation; and (d) the strain–stress relation is formulated.

Because of the extensive nature of the subject, only a limited scope of phenomena is covered. Plastic, viscous etc. time-