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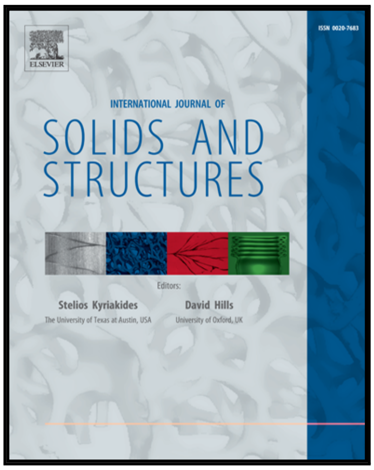

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The counterintuitive out-of-plane strength of incompressible orthotropic hyperelastic materials

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Abstract

Constitutive models for incompressible orthotropic hyperelastic materials are of interest here. We first consider the corresponding linearised theory which has six independent elastic moduli and examine its response in simple tension. Expressions for the Poisson's ratios and the ratios of the three Young's moduli are obtained in terms of derivatives of the strain-energy density in the undeformed configuration. The positive semi-definiteness of the linear strain-energy imposes restrictions on these derivatives, which are advocated here as being necessary conditions for physically realistic response of the non-linear model. Additionally, it is proposed that two further restrictions are desirable, restrictions that ensure that the out-of-plane Young's modulus is always smaller than the Young's moduli in the plane of the fibres. Since the constitutive laws for orthotropic materials are complex, simplifications in the general form of the strain-energy function are often made for mathematical tractability, the most common being strain energies that depend only on the I_1, I_4, I_6 invariants. It is shown that these reduced models do not satisfy the relative stiffness inequalities, even though they can satisfy the positive semi-definiteness conditions. This gives rise to rather counterintuitive behaviour. Firstly, the out-of-plane Young's modulus for infinitesimal deformations is always larger than at least one of the in-plane moduli and is greater than both for fibre angles that are close to orthonormal symmetry. For these fibre angles, it is also shown that the out-of-plane Young's modulus can be orders of magnitude larger than the in-plane moduli and that as the fibres are stiffened, the out-of-plane modulus is increased proportionally more than the in-plane moduli. Since the defining characteristic of fibre-reinforced composite materials is that they should be much stiffer in the plane of the fibres than out